

# Production Linkages & Volatility in an Economy:A Network based Exploration

Thesis

submitted to

Ashoka University

in partial fulfillment of the requirements for the  
M.A. Economics degree

by

Ashish Kumar



**ASHOKA**  
UNIVERSITY

Ashoka University  
Rajiv Gandhi Education City,  
Sonapat-131029, INDIA

May, 2020

Supervisor: Bhaskar Dutta

© Ashish Kumar 2020

All rights reserved

# Certificate

This is to certify that this dissertation entitled “Production Linkages & Volatility in an Economy:A Network based Exploration ” towards the partial fulfilment of the M.A. Economics degree at Ashoka University, represents study/work carried out by Ashish Kumar at Ashoka University, under the supervision of Bhaskar Dutta, Professor, Ashoka University, during the academic year 2019-2020.

Bhaskar Dutta



This thesis is lovingly dedicated to my mother **Prem Lata**.



# Declaration

I hereby declare that the matter embodied in the report entitled “Production Linkages & Volatility in an Economy:A Network based Exploration ” are the results of the work carried out by me at the Ashoka University, under the supervision of Prof. Bhaskar Dutta and the same has not been submitted elsewhere for any other degree.For chapter two, I gratefully acknowledge collaboration with and supervision of Profs. Anindya S. Chakrabarti (IIM-A), Anirban Chakraborti (JNU) and Tushar Nandi (IISER Kolkata).

Ashish Kumar





# Acknowledgments

First and foremost, I am incredibly grateful to my advisors: Prof. Bhaskar Dutta, Prof. Anirban Chakraborti, and Prof. Anindya Chakrabarti. This thesis would not have seen the light of the day without their invaluable guidance. I am particularly thankful to Prof. Bhaskar Dutta, for the productive discussions and the kind words that inspired me and kept me motivated throughout the journey of my thesis. I owe a great debt of gratitude to Prof. Anirban Chakraborti for his guidance, perspective, encouragement, and kindness that he has bestowed on me right from the days of my ICTS visit up until now. The memory of time spent along with his loving family would cherish me for my entire life. I am deeply grateful to Prof. Anindya Chakrabarti for the thoughtful discussions, giving a meaningful direction to my work, and streamlining my thought process along this journey. I am thankful to Prof. Tushar Nandi for providing me the data-set that has been essential to my thesis. No amount of words can express my gratitude to Anindita mam for welcoming me warmly into her home and treating me like her family during my time at JNU. I would also like to thank Dr. Hirdesh Kumar Pharasi and Dr. Kiran Sharma for their assistance and guidance in the course of this journey. I owe a great deal to Dr. Sunil Kumar for being a mentor and a friend right from the days of ICTS. I cherish the wonderful discussions and debates; I have had with my comrade Hrishidev along with his excellent support for my work. I especially thank Nitesh Kumar Singh, Parth Patel, and Divesh Pandey for the evergreen stimulating discussions and their rock-solid support and to all my batchmates at Ashoka for enriching the intellectual environment surrounding me. I am forever grateful to Ashoka University and its brilliant faculty for empowering me with the most peaceful weapon of knowledge. I also acknowledge the support received from ICTS-TIFR in shaping my research interests and Prof. Somdatta Sinha for giving me a great opportunity. Finally, I thank my mother, Prem Lata, whose lifelong sacrifices have made my life possible.



# Abstract

This thesis is a collection of two chapters, with a common aim to investigate the role of production networks in shaping the macroeconomic outcomes of an economy. In the first chapter, we review some of the recent developments in theoretical and empirical literature in macroeconomics that is extensively taking the aid of network theory to extend the scope of its results. More specifically, the main aim would be to revisit some of the essential results that highlight the role of production networks in transforming idiosyncratic shocks to aggregate fluctuations and the nature of the propagation. This view has mainly gained pace after the advent of the financial crisis of 2008, which essentially brought forward the significance of interdependence and interconnectedness of economies both within and across countries. This topic is of particular significance to maintain stability in an economy as it provides a theoretical framework which then could be complemented with recent developments in econometrics to model the volatility in an economy. In particular, total variability of output in a country could be traced back to its micro origins, thereby segregating the roles of idiosyncratic and aggregate shocks in causing fluctuations. Finally, we also explore a new promising avenue of decomposing the aggregate volatility into aggregate and idiosyncratic shocks. The central insight lies in rooting the reduced-form statistical factor model to a structural macro-model to filter out the role of input-output linkages in inducing sectoral co-movements in an economy, therefore, segregating the real impact of common shocks and idiosyncratic shocks in causing macroeconomic fluctuations.

In the second chapter, we investigate the phenomenon of distress propagation on a complex production network. Distress propagation on complex networks depends on the local and global topological characteristics of the network. Economic networks provide an example of a system where the global topology can be traced to the individual node-level activities that involve binary link formation between pairs of economic agents. The process of link-formation might lead to emergent intermediate-level modular architecture along with micro and macro-level properties. For some networks, individual nodes can potentially affect the whole network by acting as the epicenter of distress (e.g., a virus affecting human contact networks, starting from only one node). Economic networks, on the other hand, are seen to be more immune to node-level activities and are substantially more prone to collapse due to instability in the intermediate-level or at the macro-level. In this chapter, we have ana-

lyzed a unique administrative dataset on firm-level input-output linkages across 0.14 million production units in the Indian state of West Bengal. The production unit-level data allows us to characterize the complete network at different levels of granularity and provides an unprecedented view of the linkages in the developing economies. We show that the network of clusters of firms has a multi-hub-and-spoke structure that exhibits mild disassortativity, which varies with levels of coarse-grained filtering. With the help of a simple dynamic time series model, we characterize the mechanism of distress propagation from a chosen epicenter, across the clusters. The proposed methodology allows us to identify the most vulnerable set of firms that lead to an enormous degree of destabilization in the aggregate macroscopic network.

# Contents

<b>Abstract</b>	<b>xi</b>
<b>1 Introduction and Survey</b>	<b>1</b>
1.1 Introduction . . . . .	1
1.2 Production Networks Model . . . . .	3
1.3 The Network Origins of Aggregate Fluctuations . . . . .	9
1.4 Empirical Findings . . . . .	13
1.5 Decomposing Idiosyncratic and Aggregate shocks using Structural Factor Model	15
1.6 Summary . . . . .	23
<b>2 Distress Propagation through Trade Linkages: Modular architecture of Production Networks</b>	<b>25</b>
2.1 Introduction . . . . .	25
2.2 Data and Methodology . . . . .	28
2.3 Modularity and Shock Propagation in Network of Communities . . . . .	43
2.4 Distress propagation: Results and Discussions . . . . .	45
2.5 Summary and Discussion . . . . .	47
2.6 Future Extension: Structural Factor Macro-Model at the Firm Level . . . . .	53



# Chapter 1

## Introduction and Survey

### 1.1 Introduction

Production networks- a widely recognized idea that our economic system relies on the complex web of transactions among different entities like buyers and sellers, is quite prevalent in both contemporary and classical literature. Wassily Leontief, while studying the structure of the US economy observed that *“layman and professional economist alike, practical planner and the subjects of his regulative activities, all are equally aware of the existence of some kind of interconnection between even the remotest parts of a national economy”*.

Essentially, the idea behind his view is that in a networked economy, any sort of disturbance at a firm or an industry level may spill over to other parts of the economy through trade linkages, perhaps transforming micro-shocks to aggregate fluctuations in the process. Leontief, further argues that: *“the presence of these invisible but nevertheless very real ties can be observed whenever expanded automobile sales in New York City increase the demand for groceries in Detroit... when the sudden shutdown of Pennsylvania coal mines paralyzes the textile mills in New England, and it reasserts itself with relentless regularity in alternative ups and downs of business cycles”*.

Or more recently, in the wake of Financial Crisis of 2008, the importance of this view could be grasped from the suggestions offered in the statement of Ford’s chief executive officer, Alan Mulally (2008) during congressional testimony, in which he surprisingly requested

the government to offer bail-out packages to Ford’s key competitors, General Motors and Chrysler: *“If any one of the domestic companies should fail, we believe there is a strong chance that the entire industry would face severe disruption. Ours is in some significant ways an industry that is uniquely interdependent—particularly with respect to our supply base, with more than 90 percent commonality among our suppliers. Should one of the other domestic companies declare bankruptcy, the effect on Ford’s production operations would be felt within days—if not hours. Suppliers could not get financing and would stop shipments to customers. Without parts for the just-in-time inventory system, Ford plants would not be able to produce vehicles”*. This shows how this idea is gaining pace in the contemporary period.

In this chapter, we review the recent empirical and theoretical literature, encompassing aspects of production networks concerning its role in propagating shocks and transforming idiosyncratic shocks to macroeconomic fluctuations. Although this literature dates back to classical theory, some recent developments have spurred the growth of this literature. First, some recent developments in network theory lead to the emergence of a conceptual framework along with a much extensive set of tools that can be readily deployed to encode the interconnections among the unit of analysis comprising a network. When coupled with the tools from general equilibrium theory in economics, this toolkit enables us to study the propagation of shocks from the set of firms/industries to distant parts of the economy, through the lens of the production network, in an effective way. Secondly, the emergence of new novel data sources has paved the way for quantitative and empirical analysis to otherwise theoretical questions, such as: what are the origins of aggregate fluctuations, or tracing aggregate variability to micro-shocks affecting granular economic entities. This is also visible from the Chapter 2.1 of this document, where we are leveraging a novel data-set comprising supplier-customer relations of thousands of firms, quantifying the shock propagation in modular production network using tools from network theory and econometrics.

We intend to provide a brief survey of the emerging literature that leverages the above developments, with the sole focus on the macroeconomic implications of production networks. In section 2, a simple model of production networks is presented, which serves as a benchmark to the primary theoretical literature in the rest of the chapter. The results mainly concern with the role of production networks in inducing fluctuations at an economy-wide level. In section 3, some of the seminal results have been discussed, providing conditions under which micro-level shocks would be translated into aggregate fluctuations using the mechanism of



input-output linkages. In section 4, we review some studies, which deploy, quantitative and empirical tools, to quantify the role of production networks in the propagation of shocks at the firms or industry levels. In section 5, we discuss the approximate reduced form factor model rooted in a structural macro model that explicitly considers the role played by trade linkages in spreading the idiosyncratic shocks to distant parts of the production network. That would help in capturing the role played by sector-specific shocks in causing aggregate volatility. In section 6, we conclude by listing some of the emerging avenues to explore in this literature.

## 1.2 Production Networks Model

In this section, we discuss a benchmark model of production networks, which forms the basis to several important results that have been discussed in the chapter. This model closely follows a multi-sector general equilibrium model proposed by Long & Plosser(1983)[1], and several variants of this model have been deployed by many subsequent important papers.

### 1.2.1 The Model

Consider an economy comprising of  $n$  industries that employ Cobb-Douglas production technology. Each good produced in this economy can be consumed as a final product or, sourced as an intermediate product in the production of another good. The final product of  $i$ 'th industry can be produced by the following production function:

$$y_i = z_i \kappa_i l_i^{\alpha_i} \prod_{j=1}^n x_{ij}^{a_{ij}} \quad (1.1)$$

where hicks neutral productivity shock is denoted by the symbol  $z_i$ ,  $\kappa_i$  is a constant used for normalization,  $l_i$  represents labor deployed in industry  $i$  with  $\alpha_i$  denoting its share,  $x_{ij}$  represents the quantity of good  $j$  used as an input for producing good  $i$ , and,  $a_{ij}$  denotes the share of good  $j$  in the production process for good  $i$ . The Cobb-Douglas assumption is reflected from the fact that  $\alpha_i + \sum_{j=1}^n a_{ij} = 1 \forall i$ . The above equation shows the interconnectedness in an economy through the use of intermediate inputs in the production process. This economy also consists of a representative household with logarithmic preferences over

n goods and supply one unit of labor inelastically. The utility function is shown below:

$$u(c_1, c_2, \dots, c_n) = \sum_{i=1}^n \beta_i \log\left(\frac{c_i}{\beta_i}\right) \quad (1.2)$$

where  $c_i$  represents the amount of the good  $i$  consumed and  $\beta_i$  denotes the share of that good in the consumption basket of the consumer. The equilibrium characterization of this model is standard. All the firms maximize their profits, households maximize their utility and all the markets clear in an equilibrium. The price and quantity profile corresponding to the situation when all three conditions are satisfied constitutes an equilibrium.

The input-output matrix representing the trade linkages between  $n$  industries is denoted by:  $\mathbf{A} = [a_{ij}]$ . Any typical element  $a_{ij}$ , denotes the share of input  $j$  used in producing output in  $i$ 'th industry. Since, for every industry, we are assuming a Cobb-Douglas technology and coupled this with the fact that  $\alpha_i > 0$  for all implies that  $\mathbf{A}$  have row-sums strictly less than one and is an element-wise non negative matrix. This in turn guarantees that, the absolute value of the largest eigenvalue is less than 1. A special measure called Domar-weights are defined to capture the industry  $i$ 's share of total GDP.

$$\lambda_i = \frac{p_i y_i}{\text{GDP}}$$

Finally, the Leontief inverse of an economy is represented by  $\mathbf{L} = (I - \mathbf{A})^{-1}$ . Notably, this could also be expressed as  $\sum_{k=0}^{\infty} \mathbf{A}^k$ . This measure illustrates that a typical element  $l_{ij}$  of  $\mathbf{L}$  matrix, measures the importance of industry  $j$  as an indirect and direct supplier to industry  $i$ . To see this, take  $i \neq j$ , then above term implies that  $l_{ij} = a_{ij} + \sum_{k=1}^n a_{ik} a_{kj} + \sum_k^n \sum_m^n a_{ik} a_{km} a_{mj} + \dots$ , where first term simply reflects the importance of the  $j$  as direct supplier to  $i$  and every subsequent term describes the importance of  $j$  as a supplier to  $i$ 's suppliers and so on. This term is often dubbed as Katz-Bonacich centrality measure of agents. In other words, it accounts for all the directed paths of varied lengths connecting industry  $j$  to industry  $i$  in the given production network. The equilibrium in this setup leads to some important results described below as shown in Acemoglu(2012)[2]:

**Theorem 1.2.1.** *The log output of industry  $i$  is given by:*

$$\log(y_i) = \sum_{j=1}^n l_{ij} \epsilon_j + \delta_i \quad (1.3)$$

where  $\delta_i$  represents a shock-independent constant.

The above theorem is profoundly crucial in highlighting the role of interlinkages in transmitting shocks across different industries. Equation 1 clearly shows that output in any industry  $i$  is dependent on the productivity shocks realized in industry  $j$ , and the channel is the trade linkages between them. Note that, since the above expression is dependent on the Leontief Inverse, it reflects that not only industries are not immune to shocks from their direct suppliers, but also any shocks to indirect neighbors can also affect the firm. Lastly,  $l_{ij}$  in the above expression, also means that the productivity shocks propagate downstream and travel from direct and indirect suppliers to their respective customers.

**Theorem 1.2.2.** *The economy's (log) real value added is given by:*

$$\log(GDP) = \sum_{i=1}^n \lambda_i \epsilon_i \quad (1.4)$$

where:

$$\lambda_i = \frac{p_i y_i}{GDP} = \sum_{j=1}^n \beta_j l_{ji} \quad (1.5)$$

Theorem 1.2.2 characterizes the role of each industry in terms of its impact on the volatility of total output. In particular, total log-output is a linear combination of the weighted productivity shocks in all the industries, where the weights are expressed in terms of the Domar-weight of that respective industry. Therefore, the Domar-weight of industry  $i$  is a sufficient statistic characterizing the impact of industry on total output. Thus, it turns out that in a Cobb-Douglas set-up, the weight of an industry is directly proportional to the industry's share of output in the total GDP. Notably, the Domar weight is also dependent upon the Leontief Inverse (see Equation 1.5), which means that the production network has a role in determining its value. Theorem 1.2.1 is essential behind the intuition of this result, as it establishes the downward propagation of shocks from industry to its direct and indirect customers. Therefore, disturbances experienced in industries that are prominent input-suppliers to the rest of the network (i.e. economy) would have a more pronounced effect on the aggregate fluctuations.

**Demand-Side Shocks** In the above discussion, the results are based on the productivity shocks experienced by the industries, which are generally supply-side shocks. Acemoglu(2016a)[3], introduces government demand in the above framework and then carries the same analysis with regard to the demand shocks. This modification is reflected in the market clearing condition as stated below:

$$y_i = c_i + g_i + \sum_{j=1}^n x_{ji}$$

The change in government demand is then treated as an exogenous demand shock. Again, solving for equilibrium leads to the following result:

**Theorem 1.2.3.** *The output of industry  $i$  is given by:*

$$y_i = \sum_{j=1}^n l_{ij}g_j + \left(1 - \sum_{k=1}^n\right) \left(\sum_{j=1}^n l_{ji}b_j\right) \quad (1.6)$$

where:

$\mathbf{L} = (\mathbf{I} - \mathbf{A})^{-1}$  is the economy's Leontief inverse

$g = (g_1, \dots, g_n)'$  is the vector of government purchases from  $n$  industries

In contrast to theorem 1.2.1, where the channel through which shock in industry  $j$  affects output in the industry  $i$ , is captured by the term  $l_{ij}$ , but now, the effect of a demand shock of  $j$  on  $i$  is captured through  $l_{ji}$ . It simply means that the demand shocks propagate upstream while supply-shocks propagate downstream, through direct and indirect neighbors. To see this, suppose the demand for the product of industry  $j$  increases (a positive shock), then  $j$  would start demanding higher inputs from its suppliers-a positive shock to  $j$ 's suppliers. Similarly, all the demand shocks would propagate upstream. To conclude, the above model rests on some assumptions, like Cobb-Douglas technologies and preferences, etc. In the next section, we will briefly look at some results derived through relaxing some assumptions.

## 1.2.2 General Production Technologies & Extensions

By changing the assumption of Cobb-Douglas technologies, on which the baseline model rests, we will see how this alters the patterns of shock propagation. One of the consequences of the Cobb-Douglas assumption is that, any industry in the above model would keep the

proportion of its expenditure across different inputs to be same, even after facing a shock. But some studies, like Carvalho(2016)[4],Baqaee & Farhi(2018)[5] shows that when this assumption is relaxed, it leads to richer propagation patterns across production networks. In particular, both these papers replace the production function specified in equation 1.1 by a structure involving nested constant elasticity of substitution (CES). Although, a closed form solution is not possible, but they found out using first-order approximations that, a shock hitting industry  $i$  affects rest of its neighbors and other parts of the network via two distinct mechanisms. First, as earlier, a shock hitting industry  $i$  propagates downstream to other industries. This is common in both the frameworks. In addition to this channel, a productivity shock can also lead to reallocation of resources across all industries induced by the elasticities of substitution between inputs. For instance: a negative productivity shock to industry  $i$  leads to an increase in price of good  $i$ , which could result in distracting some of  $i$ 's customers to industry  $j$ , when  $i$  and  $j$  are substitutes. Thus, propagation patterns in a general technology does not remain confine to only downstream propagation.

### 1.2.3 Hulten's Theorem

In theorem 1.2.2, Domar weight of an industry was shown to be sufficient statistic to capture the impact of any productivity shock to that industry on overall economy, under special conditions of cobb-douglas technology. However, Hulten's theorem [6], establishes something more general. It states that, in any efficient economy, the impact on total output of a total factor productivity shock hitting an industry would be captured by the Domar statistic upto a first order approximation. Thus, it can be written as:

$$\frac{d\log(GDP)}{d\log(z_i)} = \lambda_i \tag{1.7}$$

where:

$z_i$  : TFP shock to industry  $i$

$\lambda_i$  : Domar weight corresponding to industry  $i$

Many studies have taken the aid of this theorem to establish the role of granular entities in causing aggregate fluctuations (Gabaix(2011)[7], Carvalho(2016)[8]). But at the outset, this result may seem counter-intuitive: How come any industry's role in contributing to

aggregate fluctuations is limited to its relative size in an economy, disregarding any role for its position in the network? The key to this dilemma is the condition under which Hulten’s theorem holds. Since this theorem holds only for an efficient economy, therefore, equilibrium efficiency and envelope theorem are right at the heart of this theorem. To fix ideas, suppose that an industry  $i$  face positive productivity shocks. Then, this affects the equilibrium outcome in two distinct ways. First, due to a positive shock, the production possibility frontier of the economy shifts outwards. In addition to that, this may lead to a reallocation of resources across different sectors in an economy. But when an equilibrium outcome is efficient, the effect via the second channel of resource allocation is second-order (by envelope theorem), and could be safely ignored in the first-order approximation. This then implies, Hulten theorem may not hold when equilibrium outcomes are inefficient, and in these cases, the production network becomes salient and would have a bite in determining equilibrium.

### 1.2.4 Frictions and Market Imperfections

As, Hulten’s theorem does not hold in the wake of imperfect competition, the nature of propagation patterns differ widely depending upon the extent of imperfections being considered in the model. In the baseline model, and, the subsequent results assumed perfect competition. But several studies have considered different variants of the same model, but with imperfections introduced in the form of markups between marginal revenue and marginal costs. Bigio & La’O(2017)[9] studies one such kind of models, and investigates the interactions between production network, productivity and markups. Although, they find that the equilibrium outcome is affected by the distribution of markups, nevertheless, the propagation patterns in a distorted and an undistorted economy coincide, when production process follows a cobb-douglas technology. Baqaee & Farhi (2018)[10], then analyzes the same situation using a more general CES technology, and reach to a conclusion that productivity shock’s first-order impact can be decomposed into two terms: First, accounting for the technological effect induced by the shock, and, second, accounting for the change in the allocative efficiency resulting from the shock. They also find that productivity patterns change considerably owing to distortions in the economy, when a more general production function replaces cobb-douglas technology, mainly because of the change in economy’s allocative efficiency. Grassi(2017)[11], even endogenizes the markups by allowing them to change in

the wake of productivity shocks. Therefore, this not only change the behavior of downstream propagation but also introduces upstream propagation owing to a propagation shock, mainly because of the change in demand for intermediate inputs which in turn is induced by the change in market concentration. This effect would have been absent in the model assuming exogenous wedges.

### 1.2.5 Endogenous Production Networks

In the above analysis, input-output linkages were potentially acting as a source of fluctuations in an economy and thereby reflected a mechanism of shock propagation, but so far, production networks were themselves immune to any shocks. In other words, the production networks were thought to be exogenous while conducting the analysis. This assumption may not hold, when entities in the network respond to different incentives created by shocks. For instance: Firms may want to build new connections with another firm experiencing positive productivity shocks, to reduce their input costs, etc. A small literature has emerged, which is trying to model such phenomena. Atalay(2011)[12] proposes a model, wherein nodes are forming links with other agents based on the preferential treatment. That simply means that nodes which are more popular in the network would attract more links. Similarly, Carvalho(2014)[13] follows a friendship search model proposed by Jackson(2008)[14], to suggest an industry-level network formation model, in which, existing input-output linkages are used to forge new connections. On the other hand, Oberfield(2018)[15], explicitly incorporates the incentives behind the formation of links in his model, which other papers abstains from modeling. That results in the emergence of some star-suppliers, selling their goods to many industries. Finally, Taschereau-Dumouchel(2018)[16], builds a firm-level network formation model, based on strong-complementarities between firms. Their main finding shows that a cascade of shutdowns among firms may result owing to the failure of some firms partly due to the inherent complementarities embedded in their model.

## 1.3 The Network Origins of Aggregate Fluctuations

In the section 1.2, discussion clearly illustrates that the trade linkages in a production network can act as a mechanism for shock-propagation in an economy. Next, we will review some

studies that specifically investigate one particular question in this regard- Can production networks generate aggregate fluctuations from microeconomic shocks experienced by disaggregated sectors/industries? This would shed light on the long-pending question of whether some part of observed macroeconomic variability could have its origin in idiosyncratic shocks realized at a disaggregated levels.

Lucas(1977)[17], in his influential essay on business cycles, downplayed the validity of the above hypothesis. The basis of his dismissal was based on an argument of diversification, which stated that, in an economy composed of  $n$  industries facing independent shocks, the volatility stemming from such shocks would be proportional to  $\frac{1}{\sqrt{n}}$ , which at a very high level of disaggregation is not a cause of concern. However, his argument completely disregards the role of input-output linkages as a medium of propagation. In that framework, comovement between sectors can occur, thus leading to aggregate fluctuations, even though the cause behind it are independent shocks.

In this section, we review some of the studies that have essentially analyzed the role of production networks in translating idiosyncratic shocks to aggregate fluctuations. In the process, they have given some general conditions under which the above phenomenon will hold. The results here ensue from the baseline model studied in Section1.2.

### 1.3.1 From Micro-shocks to Macro-fluctuations

Continuing with our earlier framework laid out in Sec 1.2, we will make some modifications to the variables. Suppose that, log productivity shocks are denoted by  $\epsilon_i = \log z_i$  and they are assumed to be independent and identically distributed with finite standard deviation  $\sigma$  and zero mean. This would ensure that economy only faces sector-specific idiosyncratic shocks. For simplicity also assume that labor share across all industries are constant; i.e.,  $\alpha_i = \alpha \forall i$ . Then the following result follows from equation 1.4, which simply establishes a relationship between aggregate fluctuations resulting from idiosyncratic shocks.

$$\sigma_{agg} = \text{stdev}(\log(GDP)) = \sigma \|\lambda\| \tag{1.8}$$

where:

$$\|\lambda\| = \left( \sum_{i=1}^n \lambda_i^2 \right)^{1/2}$$



By using the fact that  $\sum_{i=1}^n \lambda = \sum_{i,j=1}^n \beta_j l_{ji} = 1/\alpha$  and some algebraic manipulation, we can arrive at the following expression:

$$\sigma_{agg} = \frac{\sigma/\alpha}{\sqrt{n}} \sqrt{1 + n^2 \alpha^2 \text{var}(\lambda_1, \dots, \lambda_n)} \quad (1.9)$$

The above relationship helps us dismember the conditions under which the argument proposed by Lucas(1977)[17] holds and fails. Notice that, when Domar weights are identical across all industries, then the  $\sigma_{agg}$  is proportional to the  $1/\sqrt{n}$ , as proposed by Lucas. But when the distribution of Domar weights exhibit significant heterogeneity, then the fact that  $\sigma_{agg}$  depends on the variance of Domar weights results in the breakdown of the Lucas argument. This is at the heart of the analysis carried by Gabaix(2011)[7], wherein he establishes the granularity hypothesis- shocks emanating from micro economic entities can result in aggregate fluctuations if the distribution of firm sizes exhibits significant heterogeneity, hence resulting in higher level of aggregate volatility than the Lucas benchmark of  $1/\sqrt{n}$ . Indeed, he showed in his analysis that the distribution of the firm sizes in the US closely follows a Pareto distribution, evidence augmenting granular hypothesis and rejecting arguments presented by Lucas(1977)[17].

### 1.3.2 Networked view on Aggregate Fluctuations

In the previous section, we saw that the Domar weights of industries are tightly linked with their respective contribution to the aggregate volatility, hence paving one way to trace macro-fluctuations to their source. However, now we would focus on the role of trade linkages in translating idiosyncratic shocks to aggregate volatility. To do that, first we need to set the Domar weights equally across all industries, to capture the role of input-output linkages in causing fluctuations. For that, set  $\beta_i = 1/n$  for all  $i$ . This normalization results in, Domar weights reflecting the different roles of industries in the production network of the economy: i.e.,  $\lambda_i = v_i/n$ . The  $v_i$  term equals  $\sum_{j=1}^n l_{ji}$ , which measures the significance of sector as a direct as well as indirect input supplier to other sectors in the economy and is captured by the Leontief Inverse matrix as argued earlier. Now using equation 1.5 & 1.8, to establish a relationship between volatility in an economy and production network, as shown

below:

$$\sigma_{agg} = \frac{\sigma}{\sqrt{n}} \sqrt{\alpha^{-2} + var(v_1, \dots, v_n)} \quad (1.10)$$

The above equation highlights the key result obtained in Acemoglu(2012)[2], which provides novel insights in the investigation of tracing aggregate shocks to its micro-origins. In particular, equation 1.10 shows, that the volatility of an economy is directly proportional to the heterogeneity in the role of different sectors as input suppliers in an economy. So, if an industry which is an important supplier in a economy is hit by a shock, then it could result in substantial amount of volatility at macroeconomic level, due to wider dispersion of shocks through the channel of production network. This result completely opposes the diversification argument by linking the asymmetric positions of different sectors in a production network to the overall fluctuations. More specifically, Acemoglu(2012)[2] gives conditions on first-order and second-order degree distributions of the sectors in a network, which would lead to overall volatility greater than the  $\frac{1}{\sqrt{n}}$ . Those conditions generally corresponds to a fat-tailed distribution of suppliers, thus illustrating the widely dispersed role of sectors in an economy. The main intuition driving the result can be grasped by looking at the Leontief matrix which is at the center-stage of this result. As argued earlier, it closely corresponds to the Bonacich-centrality measure of the network, which normally captures the centrality of different nodes in a network, with regard to their reach to distant parts of the network. In particular, a sector is more centrally positioned in the production network, not only if itself is an important supplier to other sectors, but also if it is an input supplier to other central sectors in the network. This kind of behavior also highlights the small-world nature of the production network, wherein most sectors are not trading with each other nevertheless, they are only few links away from any sector in the network due to the existence of some general-purpose suppliers, thus signifying the idea that production networks can act as a mechanism to propagate idiosyncratic shocks from one part of the network to other and causing aggregate fluctuations along the way.

## 1.4 Empirical Findings

Until now, we used a simple model of production networks to illustrate some of the theoretical results concerning the role of production networks in shaping aggregate outcomes. Different papers have used variants of the stated model to establish theoretical results. In this section, we review some of the advances made along the empirical dimension in quantifying the role of production networks in shock-propagation.

### 1.4.1 Evidence at the Industry-Level

Acemoglu(2016)[3], builds a slight variant of the model shown in section 2 of this chapter. The main exercise carried out in the paper, is to quantify the propagation effects of an idiosyncratic shock hitting an industry, under the lens of a network structure. To conduct their analysis, they focus on a specific relationship between the volatility in growth rates of a sector as a function of own TFP shocks and other industries shocks, affecting it through the channel of the production network. This relationship is evident from the equation specified below:

$$\Delta \log(y_i) = \underbrace{\Delta \epsilon_i}_1 + \underbrace{\sum_{j=1}^n (l_{ij} - \mathbb{1}_{j=i}) \Delta \epsilon_j}_2$$

where:

$\mathbb{1}$  : Indicator function

The above equation illustrates that a sector specific volatility is also induced by the propagation of shocks through network effects (2nd component), apart from, originating from its own idiosyncratic shocks (1st component). Acemoglu(2016)[3] complemented the input-output data with detailed sectoral output data consisting information related to the productivity of the industries. This enabled them to recover the sector-specific productivity shocks and through input-output linkages, they were able to trace the effect of those shocks on other parts of the network. To capture the effect of productivity shocks, they deployed distinct versions of the regression equation specified below:

$$\Delta \log(y_{it}) = \delta_t + \psi \Delta \log(y_{it-1}) + \beta_{own} Own_{it-1} + \beta_d Downstream_{it-1} + \beta_u Upstream_{it-1} + \varepsilon_{it}$$

where  $\text{Downstream}_{it-1}$  variable captures the weighted average of the shocks affecting sector  $i$ 's direct and indirect suppliers in previous period, where weights are given by the entries of the Leontief matrix. Similar, relationship is shown between  $i$  and his customers through  $\text{Upstream}_{it-1}$ . This framework effectively traces the impact of productivity shocks on the entire network. The evidence, shows that the downstream propagation of the productivity shock is both statistically and economically significant, a fact consistent with the theory. Additionally, no impact of productivity was evident on the upstream propagation, again matching with the theoretical result. On contrary, when Acemoglu(2016)[3], considered the demand shocks in the form of increased government demand, they found upstream propagation to be statistically and economically significant. Although, this empirical framework provides a valuable tool to quantify the contribution of micro-shocks, nevertheless, endogeneity concerns lingers around, especially with regard to the TFP growth as an endogenous variable. Availability of more granular data, can account for these concerns and some studies have been carried out in this regard, as discussed in the next section.

### 1.4.2 Evidence at the Firm-Level

In this section, we review some studies that have carried out the empirical analysis using firm-level input-output linkages. Not only is this data more granular, but also rich in important aspects, especially to capture the salience of production networks as a mechanism to propagate idiosyncratic shocks across the economy. The main novelty of the firm-level data lies in its capability to identify the real underlying propagation mechanism, simply because actual propagation happens at the level of firms. Secondly, actual shocks are realized at the firm level, thereby this data gives a promising avenue to recover exogenous shocks (such as localized natural disaster or a labor strike) coupled with the related variation that arise in response to such shocks. Therefore, endogeneity concerns haunting industry-level analysis, could be overcome by deploying more granular data involving production networks. Several studies, are leveraging different sources of this rare data. Barrot(2016)[18], in his influential work combines firm-level data along with the timing and locations of the natural disasters to account for the propagation across the firm-firm network in context of USA. In particular, they looked at the impact of a firm's experience of a natural disaster, on its immediate customers and suppliers, by regressing change in quarterly sales value of firms on a dummy variable indicating whether that firm's suppliers were present in the region struck with a

natural disaster. They document that customers of the affected firms, saw a drop in their quarterly sales of almost 2-3%, therefore establishing the salience of firm-level trade linkages. On the similar vein, Boehm et al.(2019)[19], tries to capture the propagation patterns across the US firms, induced by the earthquake affecting Japan in the year 2011. Their estimates also found a sharp response in the output of US-affiliates of Japanese multinational corporations. A common evidence linking both these studies, suggests that a shock propagates to a firm's immediate supplier and customer, but to establish that firm-level trade linkages can lead to sizeable aggregate fluctuation, evidence is needed to support the claim that propagation affects distant parts of the production network as well. Carvalho(2016)[4], provides evidence in this regard by leveraging a rich data-set comprising of the entire universe of firms in Japan. More specifically, Carvalho(2016)[4], traces the disruption caused by the Tsunami and the earthquake experienced by Japan in 2011, throughout the production network. They found substantial evidence of reduced sales growth of the firms whose suppliers were based in disaster-areas. In addition, they also found evidence of both downstream and upstream propagation to customers and suppliers of the firms belonging in the disaster struck region. The direct propagation coupled with the indirect propagation indicate that regional disturbances could lead to aggregate volatility. Finally, Demir et al.(2018)[20] exploits a novel VAT firm to firm transactions data of Turkey to provide evidence that an unexpected financial shock in the form of a tax on trade, resulted in propagation of shock across the production network, while causing aggregate fluctuations in the economy.

## **1.5 Decomposing Idiosyncratic and Aggregate shocks using Structural Factor Model**

### **1.5.1 Introduction**

In this section, we provide a formal economic model to understand the mechanism of shock propagation in an input-output network of firms instead of sectors, as has been carried out in the literature. This section closely follows the framework proposed in the paper by Foerster (2011)[21] wherein they are investigating the causes behind the high variability observed in the sectoral output. Not only is sectoral output highly variable but also cross-correlated. Some of the essential questions that prop up are: What are the origins of business cycle

fluctuations? Do idiosyncratic shocks-micro shocks realized at individual firms or industries-have any significant role in causing short-run volatility at the macroeconomic level? Or are the common shocks hitting all the industries a predominant force behind those fluctuations? Foerster (2011)[21], tries to answer the above and some other related questions systematically. In particular, they construct and estimate a multi-sector dynamic general equilibrium model, allowing for both sector-specific idiosyncratic shocks as well as common shocks to induce aggregate volatility. The evidence from the paper suggests that sectoral shocks account for a considerable proportion of the variation observed in aggregate output growth, especially after the great moderation: a period of low volatility in output growth of the USA (1984-2007). Therefore, this evidence augments the theory of production networks-proponent of the view that trade linkages could amplify the industry-specific shocks manifold by propagating shock emanated from one node to distant parts of the network, resulting in aggregate volatility.

Much of the challenge lies in identifying and isolating the share of common shocks and idiosyncratic shocks from the observed volatility. The difficulty lies the fact that part of idiosyncratic shocks gets shrouded in the common shock, partly because it travels through trade linkages across distinct parts of the network and the resulting co-movement between the volatility of the sectors appears to be caused by some aggregate shock, even though the exact mechanism at play is that, a networked economy has translated a sector-specific shock to an economy-wide disruption. The novelty of the Foerster (2011)[21] paper lies in disclosing this exact mechanism by rooting the evidence using a structural model. The Foerster (2011)[21] paper falls into pervasive literature on multi-sector real business cycle models. These models were first introduced in Long and Plosser(1987) [22], Forni and Reichlin(1998) [23], and, Shea(2002) [24] wherein factor analytic models were used in conjunction with the framework of input-output linkages to decompose the volatility. The evidence reported in these papers suggests that idiosyncratic shocks to sectors or industries possess the potential to generate aggregate fluctuations. Another related strand of literature has studied this issue from a theoretical perspective. Long & Plosser(1983) [1], Horvarth(1998,2000) [25, 26], Dupor(1999) [27], and, Carvalho(2007) [28], are some of the well-known papers to build structural calibrated models to study the concerned phenomenon. These papers explicitly took into account the networked interactions in an economy captured by its input-output matrix and then provide some overall results along with the predictions about the volatility, that a single constituent in a system can generate, which further is essentially dependent upon the nature of linkages that are being defined in their respective models. For instance: Long & Plosser(1983) [1] paper limits its scope by disallowing the usage of

capital in a multi-sector framework of an economy. Carvalho(2007) [28], studies a version of the above model, but still without including capital in the production process of industries in his model. However, Horvarth(1998) [25] and Dupor(1999) [27], both in their papers have kept the environment of the model more general by allowing trade interactions within sectors at two levels-by allowing sectors to source the inputs from other sectors that are readily used for the production of their final products and secondly, they also allow sectors to source the final goods from other industries that are then deployed in the production processes of their capital goods. Despite this addition, they only analyze the limiting case of full capital depletion in a single period, thereby eliminating the supply chain that could have emerged due to capital goods.

To summarize, essentially, there are two views predominant in the literature concerning the role of sector-specific shocks and aggregate shocks in causing aggregate volatility. One side argues in favor of reduced-form statistical factor models to decompose the share of both types of shocks in the observed total volatility. In contrast, the other side emphasizes more upon rooting their analysis in theoretical models by carefully specifying the nature of interactions embedded in an economy.

Foerster (2011)[21] is appreciative of the views stemming from both the camps and, therefore, tries to reconcile their methods and findings to give something more general. In particular, Foerster (2011)[21] developed a methodology that allowed to recover the underlying sector-specific idiosyncratic shocks using the industrial output growth data. In particular, it filters out the part of the total variation arising due to idiosyncratic shocks using a dynamic general multi-sector model. Then using an approximate factor model as a reduced form of the structural model, it estimates the share of common factors/shocks driving the aggregate volatility observed in an economy. Notice that this exercise eliminates the concern of upward bias in the concerned estimate as, sectoral comovement induced by trade linkages stemming from idiosyncratic shocks has already been filtered out, therefore improving upon the simple reduced-form statistical models. The evidence reported in the paper clearly shows that post-1983, the role of aggregate shocks in inducing fluctuations in the total output growth rate followed a steep decline, which, therefore, implies that idiosyncratic shocks were a dominant force behind aggregate fluctuations, thereby strengthening the granular hypothesis. More specifically, the share of sector-specific shocks rose from 20% for the volatility in output growth in the 1972-83 period to nearly 50% for the volatility in the 1984-2007 period. Although, Foerster(2011)[21] made a significant stride in highlighting that

a nontrivial fraction of aggregate fluctuations can be traced back to idiosyncratic shocks, but still, they deploy Cobb-Douglas production function in their model—a simplifying assumption to make model tractable. Coupling this fact with some empirical studies showing evidence of propagation patterns making a sharp departure from the Cobb-Douglas benchmark, thus raises concerns about the quantitative accuracy of the results in their paper. Atalay(2017)[29], rectify the above deficiency, by using CES technologies and preferences and then calibrating a similar structural model to arrive at an approximate factor model as reduced form. The complementarity among inputs induced by CES technology results in a more pronounced role for microeconomic shocks to contribute to aggregate volatility than the baseline case of Cobb-Douglas technologies would suggest. Furthermore, as evidenced in Atalay(2017)[29], about 83% of the variability observed in overall output growth is attributable to these microeconomic sector-specific shocks.

In the rest of the section, we would briefly describe both the empirical-strategy and theoretical model used in Foerster(2011)[21]. In section 1.5.2, we would discuss the statistical factor model to decompose aggregate volatility and the drawbacks of this method. Then, in section 1.5.3 discussion would revolve around on how to arrive at an approximate statistical factor model rooted in a structural model of an economy. In Chapter 2.1, we would briefly touch upon the issue of implementing this method using a firm-level production network.

## 1.5.2 Statistical-Factor Analysis

As described earlier, Forni & Reichlin (1998)[23], brought the idea of an approximate factor model to the mainstream macroeconomics. They propounded to use this method to model the covariance matrix of sectoral production. This method entails estimating a factor model on a panel of sectoral growth rates ( $N \times 1$ ),  $\Delta \log y_t = X_t$ :

$$\Delta \log X_t = \Lambda F_t + u_t \tag{1.11}$$

$F_t$  : vector of latent factors of low dimension

$\Lambda$  :  $N \times k$  coefficient matrix representing factor loadings

$u_t$  :  $N \times 1$  vector representing industry-specific shocks



The classical factor analysis assumes,  $F_t$  and  $u_t$  to be i.i.d sequences of random variables and they both are uncorrelated to each other. Also,  $u_t$  has a diagonal covariance matrix. It simply means that  $X_t$  is an i.i.d. sequence of random variables with covariance matrix given by:  $\Sigma = \Lambda \Sigma_{FF} \Lambda' + \Sigma_{uu}$ , where  $\Sigma_{FF}$  and  $\Sigma_{uu}$  are covariance matrix of  $F_t$  and  $u_t$ , respectively. By assumption,  $\Sigma_{uu}$  is diagonal, which implies that common factors are the underlying reason behind any evident covariance between  $X_t$  and  $F_t$ . Approximate factor models, on the other hand rely on the asymptotic properties and weaken the above assumptions by allowing  $u_t$  to satisfy weak cross sectional dependence. They deploy penalized least squares criteria to consistently arrive at the number of factors. They also use the above method to estimate factors by estimating principal components.

However, all this analysis rests on the crucial assumption that most of the comovement relies on the common factors. Nevertheless, this setup completely overlooks the fact that production networks can induce significant correlations purely from idiosyncratic shocks, which would then result in an upward bias in the estimate of aggregate shock's share in total comovement. In other words, what might appear to be a common shock to an econometrician may be the result of the endogenous comovement generated by the interactions between sectors in a production network. To account for this fact, one must ideally take the approach of rooting the approximate factor model in a structural model, which entirely takes into account the role of these input-output linkages.

### 1.5.3 Structural Factor Analysis: The Model

The model described in the previous section rests on the crucial assumption that the covariance matrix of  $u_t$  satisfies weak cross-sectional dependence. However, the presence of trade linkages between the sectors could generate comovement across sectors. This transformation of idiosyncratic shocks to common shocks in driving the variability of output by trade linkages would violate the above assumption. In this section, a reduced form factor approach of 1.11 would be derived from a structural model initially given by Long & Plosser (1983) [1] to alleviate the concerns regarding simple reduced form factor model. The structural model presented here is general enough to include interconnectedness in an economy at two levels: one that each sector source its material from some other sector and second that production of capital goods in a particular sector also uses the output of other sectors, a remarkable generalization of the models present in the literature. Therefore, this not only allows the

dissemination of sector-specific shocks through typical trade linkages but also allows for the possibility of distress to travel through the interlinkages for capital goods, which are more persistent in the network under a standard depreciation rate, hence a severe cause of aggregate fluctuations.

In this section, we introduce a dynamic multi-sector general equilibrium model used in Foerster(2011)[21]. We will briefly present the complete process to characterize the structural model in terms of a reduced form factor model, which then can be used to quantify the comovement between sectors arising from the aggregate shocks. The purpose is to show that the same exercise can be carried out using the firm-level production network, in contrast to the sector-level production network deployed by the Foerster(2011)[21].

Suppose an economy consists of  $N$  different sectors of production indexed by  $j = 1, \dots, N$ . A sector  $j$  produces a quantity  $Y_{jt}$  at date  $t$  using labor  $L_{jt}$ , capital  $K_{jt}$ , and, other intermediate level of inputs sourced from others sectors,  $M_{ijt}$  using a Cobb-Douglas technology as specified below:

$$Y_{jt} = A_{jt} K_{jt}^{\alpha_j} \left( \prod_{i=1}^N M_{ijt}^{\gamma_{ij}} \right) L_{jt}^{1-\alpha_j-\sum_{i=1}^N \gamma_{ij}} \quad (1.12)$$

where  $A_{jt}$  : productivity index for sector  $j$

The first source of interconnectedness is evident from the production function described in 1.12. More generally, an input-output matrix for this economy can be represented using  $N \times N$  matrix  $\Gamma$  with an element representation of the form  $\gamma_{ij}$ , which simply means the supply of materials from sector  $i$  to sector  $j$ . Then the sum of rows of  $\Gamma$  typically means the significance of the sector as the supplier to other sectors. On the contrary, a column sum of  $\Gamma$  means the input-share of other sectors material in the output of a sector. In other words, row sums capture the out-degree of the sector as a supplier, and column sums measure the in-degree of a sector as a customer.

Suppose,  $\alpha = (\alpha_1, \alpha_2, \dots, \alpha_n)'$  is the vector of capital shares. Also, let  $A_t = (A_{1t}, \dots, A_{Nt})'$  represents productivity index of different sectors in a vector form. Suppose,  $\ln A_t$  follows the random walk process:

$$\ln(A_t) = \ln(A_{t-1}) + \varepsilon_t \quad (1.13)$$

where  $\varepsilon_t = (\varepsilon_{1t}, \dots, \varepsilon_{Nt})'$  represents a stochastic process with expectation to be zero, conditioned on past values with a covariance matrix  $\Sigma_{\varepsilon\varepsilon}$ . In the extreme case, when aggregate shocks are absent, then  $\Sigma_{\varepsilon\varepsilon}$  would be a diagonal matrix with diagonal entries representing idiosyncratic shocks. Otherwise, non-zero off diagonal terms of  $\Sigma_{\varepsilon\varepsilon}$  will reflect the common shocks affecting the economy.

The law of motion specifying the evolution of capital in each sector is described below:

$$K_{jt+1} = Z_{jt} + (1 - \delta)K_{jt} \quad (1.14)$$

where  $Z_{jt}$  : Investment in sector j

$\delta$  : depreciation rate

As mentioned earlier, sectors also purchase goods from other sectors to produce the capital goods required in their production process, which reflects another level of interconnectedness in the model. Now this interaction can be represented using  $\Theta$  with a typical entry denoted by  $\theta_{ij}$ . Again, the row sums of this matrix represent the importance of each sector's output as a source of investment for other industries, and column sums represent the share of other industry's output in the capital composition of a sector.

The utility function of a representative consumer as a function of products from N sectors as well as the the labor supplied is described below:

$$\mathbb{E}_\circ \sum_{t=0}^{\infty} \beta^t \sum_{j=1}^n \left( \frac{C_{jt}^{1-\sigma} - 1}{1-\sigma} - \psi L_{jt} \right) \quad (1.15)$$

Each sector also needs to obey a resource constraint as stated below:

$$Y_{jt} = C_{jt} + \sum_{j=1}^N M_{jit} + \sum_{i=1}^N Q_{jit} \quad (1.16)$$

The model yields a steady-state equilibrium which is analytically tractable. Linear approximation of the model's first order conditions and resource constraint around steady-state gives us an ARMA(1,1) model for sectoral growth,  $X_t = [\delta \ln(Y_{1t}), \dots, \delta \ln(Y_{Nt})]$ :

$$(I - \phi L)X_t = (\Pi_0 + \Pi_1 L)\varepsilon_t \quad (1.17)$$

where  $L$  represents lag operator and  $\phi$ ,  $\Pi_0$ , and,  $\Pi_1$  are  $N \times N$  matrices depending upon the model parameters,  $\alpha$ ,  $\Gamma$ ,  $\Theta$ ,  $\beta$ ,  $\sigma$ ,  $\psi$ , and  $\delta$ . Now, sectoral productivity shocks consists of both aggregate shocks as well as idiosyncratic shocks. So, we can write  $\varepsilon_t$  as:

$$\varepsilon_t = \Delta_S S_t + v_t \quad (1.18)$$

where :  $S_t$  :  $k \times 1$  vector representing aggregate shocks

$v_t$  :  $k \times 1$  vector representing idiosyncratic shocks

$\Delta_S$  : Matrix of coefficients representing how aggregate shocks affect sectoral productivities

In the above equation,  $(S_t, v_t)$  are both serially uncorrelated and mutually uncorrelated. The covariance matrix of idiosyncratic shocks is represented by  $\Sigma_{vv}$ , which is considered to be a diagonal matrix. Notice that in 1.18, linkages does not play any role in biasing the estimate. Then, by combining 1.17 & 1.18, we can write the change in sectoral growth rates in the form of factor model as described below:

$$X_t = \Delta(L)F_t + u_t \quad (1.19)$$

where,  $F_t = S_t$  and:

$$\begin{aligned} \Delta(L) &= (I - \phi L)^{-1}(\Pi_0 + \Pi_1 L)\Delta_S \\ u_t &= (I - \phi L)^{-1}(\Pi_0 + \Pi_1 L)v_t \end{aligned}$$

The above analysis provides a way to back-out a reduced form factor model from a dynamic multi-sector structural model. The common factors,  $F_t$  in the reduced model 1.19 are related to the aggregate shocks  $S_t$  in the structural model. Moreover, the idiosyncratic shocks in the reduced form model 1.19 comprises a linear combination of sector-specific shocks,  $v_t$ . However, even though elements in  $v_t$  are uncorrelated, but, trade linkages will induce correlation among elements of  $u_t$  term due to the presence of  $(I - \phi L)^{-1}(\Pi_0 + \Pi_1 L)$ . That is the reason why the reduced form model presented in 1.5.2, overestimates the presence of aggregate shocks in the total variability of output. To rectify this, we need to eliminate the role of production linkages inducing the correlations in comovement from idiosyncratic shocks while attributing the variability in total output to the aggregate shocks. The paper simply achieves that by estimating  $\varepsilon_t$  as a filtered version of the observed growth rate  $X_t$

using equation 1.17 as:

$$\varepsilon_t = (\Pi_0 + \Pi_1)^{-1}(1 - \phi L)X_t \quad (1.20)$$

Now, with 1.20, we can use a factor model to estimate the relative contribution of aggregate vs. idiosyncratic shocks without worrying for the bias that propagation effect of sector-specific shocks caused by production linkages, could create, as they have already been eliminated.

## 1.6 Summary

In this chapter, we provided a brief review of the emerging theoretical as well as empirical literature concerning the role of production networks in affecting and shaping the aggregate outcomes in an economy. We relied on a simple baseline model to highlight some of the important results, documented in different studies. The common thread binding all of these studies corresponds to the conjecture that: production networks can act as an important medium to propagate shocks across the economy and establishing the salience of production networks in translating idiosyncratic shocks to sizeable macroeconomic fluctuations. We surveyed literature, in which some studies established the theoretical plausibility of the above phenomenon, and other studies validated the theoretical findings by conducting different empirical exercises involving data representing varying levels of disaggregation. Lastly, we surveyed the literature aimed at decomposing the aggregate volatility into common shocks and idiosyncratic shocks. That not only augments the current multi-sector dynamic general equilibrium models but also complements the analysis in other areas of economics, such as international trade, growth & development, and monetary policy analysis, which would potentially like to incorporate input-output matrix to model amplification mechanism. More importantly, not only Foerster(2011)[21] paper is relevant in terms of showing the salience of input-output linkages in generating macroeconomic fluctuations; it also provides novel methodology to bridge theoretical models to statistical factor models as reduced form, that then becomes essential in settling the debate between common shocks and idiosyncratic shocks. Also, it provides an appropriate framework to carry out the same analysis at a more granular level.



# Chapter 2

## Distress Propagation through Trade Linkages: Modular architecture of Production Networks

### 2.1 Introduction

<sup>1</sup> Complex interdependent systems are often ‘robust yet fragile’ and their stability with respect to external perturbation can be seen as emergent phenomena. Both the robustness of the system and the fragility can be traced back to the nature of the network of interdependence. The mechanism of distress propagation on such complex networks in turn, is known to be dependent on the topological properties of the network, ranging from the node-level local characteristics to the network-level global characteristics[31]. Failure of individual nodes might cause cascading failures in the whole network, e.g. power grid, traffic congestions, water storage and infection spreading. The other extreme phenomena of systemic robustness with respect to localized external perturbations are regularly seen in case of economic fluctuations, e.g. economic cycles of boom and bust are known to be driven by country-level

---

<sup>1</sup>This chapter was done in collaboration with Profs. Anindya S. Chakrabarti, Anirban Chakraborti and Tushar Nandi. A revised version of this chapter has been communicated for publication. All the figures used in this chapter (except Fig 2.1) have been adopted from the same paper.[30]  
We are thankful to the Directorate of Commercial Taxes of West Bengal, India for the permission to use their data.

shocks like monetary policies, productivity fluctuations and so forth.[32]

However, this view of the economy was challenged in the aftermath of the financial crisis in 2007-09. In an important paper, Gabaix[7] showed theoretically and empirically that firm-level shocks could drive aggregate fluctuations in an economy. Following a complementary approach, Acemoglu et al. [2] showed that the phenomenon of shock propagation through sectoral linkages could impact aggregate volatility. These two crucial papers brought forward the role of *granular* economic entities in terms of explaining systemic instability. In this chapter, we provide a finer view of the network architecture of an economy, shedding light on the origin and spread of shocks and how it percolates in an interdependent economy. In particular, we combine both the approaches and show that clusters of firms that are closely connected through supply chains can be utilized to model the effects of shock propagation. In this chapter, we study the architecture of the firm-to-firm production network at different levels of coarse-grained filters and the resultant shock propagation mechanism.

Before stating the mechanism, it is useful to unpack the differences between aggregate shocks and idiosyncratic shocks. The idea of aggregate shocks[17] is quite appealing: if production cluster (say, firms or sectors) have uncorrelated disturbances, such disturbances would tend to average out as we dis-aggregate the economy into finer and finer definitions of production clusters, essentially due to the *central limit theorem*. This in turn, would imply that only aggregate shocks can have macroeconomic consequences. Indeed, the source of fluctuations in most of the business cycle literature can be traced back to aggregate supply side or demand side shocks. [26] [3],[33]. In contrast, there is new literature that emphasizes the interlinkages in the production structure of modern economies and how they might actually amplify rather than muting, the idiosyncratic effects arising out of production units.[3]

Our main proposition is that the phenomena of distress propagation is fundamentally dependent on the intermediate-level properties of the production network. In particular, the modular architecture of the network should be seen as the basic building block. Below we explain why and how it differs from the firm-level[7] and sector-level[2] granularity. First of all, we note that potential *shocks* might be local in nature affecting individual firms. A *sector* is a collection of firms that produces similar goods classified under the same type of production processes, which is more coarse-grained and granular than a firm. Firms on the other hand, might be made of plants. In an ideal world, we should trace the epicenter of



a shock at the plant-level, which is literally the smallest production unit. In general, such fine-grained data is rare and not available.

In this chapter, we have utilized an unique administrative dataset obtained from the Indian state of West Bengal, which was constructed by matching the tax-collection records of nearly 0.14 million plants while they carried out bilateral trades to purchase and sell manufactured goods. We have constructed the networks from the data collected in quarters 1, 2, 3 and 4 in the fiscal year 2016.

Our main results can be classified under three categories. One, the macroscopic properties of the production network are quite stable. The structure of firm level production network is not changing across four quarters. Therefore, the network exhibits *stickyness*. Two, there is considerable heterogeneity in the local topological characteristics of the firms' connectivities in the network. The dispersion in the number of the first-order neighbors and the second-order neighbors closely mimic the same found in sectoral data.[3] Third, the production network is modular and we construct networks of modules at various degrees of coarse-graining, where each module comprises a set of firms tightly interlinked through supply chains. Below we elaborate on each of the results.

Our results directly relate to the literature on the granular architecture of economies which started with a theoretical debate on whether granularity and sectoral interlinkages matter for aggregate dynamic properties.[33][26][28][21].The work of Gabaix[7] brought the role of granularity of firms in explaining business cycle fluctuations at the forefront. Foerster[21] modeled an economy with standard business cycle model with an explicit input-output network which was mapped directly to the industrial production and matching the model results with empirical decomposition of aggregate shocks and idiosyncratic sector specific shocks, showed evidence in favor of granularity. There is a stream of work now on utilizing observable large shocks to a set of firms or industries and tracing their impact through the input-output network; e.g. David (2013)[34] studied the impact of increased Chinese competition into the U.S. economy through input-output linkages and local labor markets, Barrot (2014)[18] and Carvalho (2016) [4] focus on the transmission of shocks arising out of natural disasters, such as 2011 Japanese earthquake, over the global input-output network.

This chapter can be viewed as a part of two strands of literature on networks in economics. First, there is a large literature on theoretical models of network formation that mimics real-world networks.[35] In our case, a particularly important class of models are

scale-free network[36] as the firm-to-firm network shows power law-like behavior in the degree distribution (both in first order and second order). The entire production network shows mild disassortativity. However, once we construct the network of communities from the production linkages, then the resulting network is assortative. Therefore, it indicates existence of multiple hubs closely connected to each other. We show that this finding has important implications for the mechanism of shock propagation. Second, there is a growing literature on statistical description of empirical networks constructed from economic and social data. This chapter provides an unique network view of the production process in developing economies. As far as we know, there is no other firm-to-firm network available in the context of developing economies.

Before getting into the analysis, we note that the main message of the chapter is the proposed link between modularity of networks and the mechanism of shock propagation. As far as we know, this is a novel way of coarsening production units and studying shock propagation as opposed to more standard ideas of coarsening at the level sectors.[3][21][33][26] This idea resonates with the literature of propagation of location-specific shocks (e.g. natural disasters[4]) through the supply chain network. However, exact causal identification of shock propagation could not be done in the present context due to limited availability of data.

The rest of the chapter is organized as follows. We first provide detailed description of data, and the methodology that went behind in the construction of the production network and backbone of that production network for both quarters for the state of West Bengal. Next, we discuss topological properties of the input-output network and all of its sub-components in both quarters. Then we analyze aggregate properties of the network, viz. assortativity and modular structure, to shed light on the nature of disaggregated network. The modular network is studied as a network of modules for shock propagation. Finally we discuss the implications of our results and conclude.

## 2.2 Data and Methodology

We have constructed the production network from quarterly tax data collected by the state officials of the state of West Bengal in India. The data has been collected under the Value-

added Tax (VAT) system introduced in India on 1st April, 2005.<sup>2</sup> In this tax collection scheme, each firm has an unique identification number. Due to the tax collection structure, both the buyer and the seller parties of a transaction are supposed to produce a record of the transaction amount and the nature of the transaction including amount of money exchanged along with the identification number of the trading partner. Then records from both sides are matched to rule out false reports of trades. Therefore a successful match between the buyer’s record and a seller’s record indicates successful completion of a trade between the buyer and the seller. In our database, that shows up as a linkage between two plants. The state officials collect and aggregate such data through tax filing once in every quarter of the year. Therefore, one can construct a network arising out of all the reported binary linkages. Timing-wise the data we have collected spans over four consecutive quarters viz. quarters 1-4, 2016. Therefore, all of our analysis in this paper has been conducted over these four snapshots of the production network.

There are about 0.14 million firms in the data, with about 0.50 million trade linkages amongst themselves in all four quarters. In the constructed network, plants are *nodes* and the trade linkages between a pair of plants represent an *edge* between the corresponding pair of nodes. We provide the summary statistics for the entire production network in table 2.1. We see from table 2.1 that the topological structure of the network is very persistent across the four quarters. Although the number of firms and trade linkages have surged between two quarters to a certain extent but overall the network is quite stable in terms of other clustering properties along with localized degree connectivity.

Table 2.1: Characteristics of production network for the Indian state of West Bengal for quarters 1-4 in the fiscal year 2016

quarter	nodes	edges	average degree	number of clusters	nodes in the giant component (GC)	edges (GC)	average degree (GC)
1	132331	510746	7.72	483	131148	510026	7.78
2	132537	510178	7.61	500	131303	509404	7.67
3	138660	541960	7.725	503	137482	541254	7.781
4	144509	582651	8.06	424	143518	582045	8.11

Columns 6-8 in table 2.1 shows that giant component or maximally connected component

---

<sup>2</sup>For the sake of completeness, we note here that this tax-collection scheme was substituted by Goods and Services Tax (GST) scheme on 1st July, 2017. However, our dataset does not overlap with the GST regime.

comprises around 99 percent of the production entities along with almost all trade linkages that were present in the original network. Note that, by construction every entity in the giant component is path connected to every other production entity in the network. Under this scenario, distress originated in any sub-graph of the original network can propagate to any other sub-graph, at least as a theoretical possibility. It is worthwhile to mention here that the edges were treated as undirected while extracting the giant component from the original network. Using this criterion trade linkages possess information on both directions: input flows (from supplier to customer) and monetary flows (from customer to supplier). As the numerical results suggest, giant components are also quite stable in terms of properties, across all four snapshots.

Table 2.2: Characteristics of the backbone ( $\alpha = 0.0001$ ) extracted from GC of the production network for the Indian state of West Bengal for quarters 1-4 in the fiscal year 2016

quarter	nodes (%)	edges (%)	number of clusters (%)	nodes in GC	average degree	normalized weights
1	19032 (14.4)	14993 (2.9)	4788	6488	2.22	0.47
2	27126 (20.4)	23920 (4.7)	5291	13446	1.759	0.38
3	29811 (21.4)	25862 (4.8)	6088	13828	1.731	0.39
4	21920 (15.6)	17492 (3)	5411	7915	2.24	0.50

### 2.2.1 Constructing Backbone of a Network

“The backbone of a network comprises of all the relevant edges at all the scales present in a system using an algorithm called **Disparity filter**.”[37]. Disparity filter falls under the category of network reduction algorithm. The essential function of this algorithm is to extract the backbone structure of the directed/undirected weighted network. Many real-world networks such as citation networks, world-wide-web networks, airport networks exhibit a fat-tailed distribution of node’s weight and strength. In those cases, the above algorithm can sufficiently reduce the dimensionality of the network while preserving the multi-scale

level of the network. The functioning of the algorithm is presented below:

**Null Hypothesis** Suppose strength of node  $i$  is represented as  $s_i$  and is expressed as:

$$s_i = \sum_j w_{ij}$$

where:

$$w_{ij} = \text{weight of link between } i \text{ and } j$$

The main attraction of the disparity algorithm is that it reduces the size of the network significantly without overlooking nodes with low strength. For that, a normalized weight  $p_{ij}$  is defined as follows:

$$p_{ij} = \frac{w_{ij}}{s_i}$$

where:

$$p_{ij} = \text{new normalized weight corresponding to link } ij$$

The algorithm first specifies a null model, under which normalized weights of a certain node with degree  $k$  is determined using the following procedure:

- (i) First,  $k-1$  pins are randomly assigned between the interval of 0 and 1. Therefore, the interval is divided into  $k$ -subintervals.
- (ii) Under the null model, the normalized weight of each link is represented by the length of the subinterval.
- (iii) The normalized weight distribution of node with degree  $k$  based on the null model follows:

$$p(x)dx = (k - 1)(1 - x)^{k-2}dx$$

**Disparity Filter** After specifying the null hypothesis and generating a normalized weight distribution based on that hypothesis, disparity filter algorithm is implemented, which is based on a  $p$ -value statistical significance test of the null model. For an actual normalized

weight  $p_{ij}$ - the p-value  $\alpha_{ij}$  corresponding to  $p_{ij}$  based on null hypothesis is given by:

$$\alpha_{ij} = 1 - (k - 1) \int_0^{p_{ij}} (1 - x)^{k-2} dx$$

which upon solving gets reduced to the following expression:

$$\alpha_{ij} = (1 - p_{ij})^{k-1}$$

Intuitively,  $\alpha_{ij}$  is the probability of having a normalized weight greater than or equal to  $p_{ij}$  under the null model. Then by specifying a significance level- $\alpha \in (0,1)$ , for any node of normalized weight  $p_{ij}$ , if  $\alpha_{ij}$  is greater than  $\alpha$ , it will be filtered out. Tuning the parameter  $\alpha$ , based on the context can eliminate irrelevant links. Thus, the backbone structure of a weighted network is extracted in an effective manner. In addition to that, the disparity filter reduces the number of edges in the original network significantly, keeping, at the same time, almost all of the weight and large fraction of nodes. As well, this filtering preserves the cut-off of the degree distribution, the form of the weight distribution, and the clustering coefficient.

**Backbone of Production Network** The production network is a large object with 0.14 million nodes and half a million edges. In order to analyze the granular nature of the network, we conduct our analysis at two levels of coarse-grained filtering. First, we have extracted the *backbone* of the giant component using *disparity filter*[37]. The algorithm extracts the backbone network by considering the relevant edges at all the scales present in the system and by exploiting the local heterogeneity and local correlations among the weights.

The filter can be tuned by a cut-off parameter  $\alpha$  that can be set exogenously, to control the number of nodes and edges that appear in the backbone. For our analysis, we kept  $\alpha = 0.0001$  and we have confirmed that our results are robust to small changes in  $\alpha$ . Table 2.2 shows basic topological features of the backbone network extracted from the giant component in quarters 1-4. This table 2.2 clearly shows that 2.9-4.8% of the trade linkages (column 3) in backbone carry 39-50% of the transactional value of the all the trade linkages (column 7) present in the entire production network. Additionally, only 14.4-21.4% of the production firms feature in the backbone in comparison to the entire network.

## 2.2.2 Network of Communities

The main motivation of this chapter is to model propagation of localized shocks through the supply chain network. The supply chain network can be constructed by identifying production clusters in the main production network. A production cluster comprises of all the firms that are very close to each other from a supply-chain perspective. To cluster firms in a statistical meaningful way, we resort to one of the standard tool from network theory toolkit called **communities**.

**Definition 2.2.1.** *Community in a Network: A community is defined as a subset of nodes within the graph such that connections between the nodes are denser than connections with the rest of the network.*

The novelty in our approach lies in the fact that we are proposing a statistically meaningful way to combine firms to form production clusters. In standard network theory set up the concept of identifying communities in a network is quite prevalent and appealing. Although there are numerous complex algorithms to identify communities; however, in its most rudimentary form, community identification simply compares the density of connections in different sub-parts of a network. The sub-graphs encompassing a higher density of interlinkages within that group, in comparison to across groups, can then be labeled as one community. That could potentially help our hypothesis, as, if within each cluster, any production plant is exposed to any kind of shock, then it would spill over to the entire cluster only by the definition of a community. We employ community detection algorithms to extract the intermediate-level properties of the network. In particular, we employ *modularity maximization* and *Infomap* algorithms, which are more coarse-grained than the backbone but reduces the dimension of the original network substantially. We construct a *network of communities* through both the modularity maximization and Infomap algorithms. Both of the methods are discussed in detail in the following passage.

### **Network of Communities obtained using Modularity Maximization Technique:**

Consider a graph with  $N$  nodes and  $L$  links that is partitioned into  $n_c$  communities, where each community have  $N_c$  nodes connected to each other by  $L_c$  links where  $c = 1, 2, \dots, n_c$ . If  $L_c$  exceeds the expected number of edges between the  $N_c$  nodes given the network's degree sequence, then the nodes of the sub-graph  $C_c$  could form a community. The modularity of

a network is then defined as the summation over all  $n_c$  communities, the difference between the links present in a partition and the expected number of links that would have been present in a randomly wired network. Maximizing modularity corresponds to finding the best community structure in a network.

$$M = \sum_{c=1}^{n_c} \left[ \frac{l_c}{L} - \left( \frac{K_c}{2L} \right)^2 \right]$$

$l_c$  = No. of links present between community  $c$

$L$  = Total No. of Links

$K_c$  = Average degree of nodes in community  $c$

The **Greedy Algorithm** community detection method uses the modularity maximization technique to partition the network in varying communities. The hypothesis of Greedy Algorithm suggests that for a given network, the partition, that has the maximum modularity corresponds to an efficient community structure. Most of the algorithms to detect communities are designed with the set objective of identifying the partition with the largest modularity. So, the best possible partition could be identified by checking  $M$  (in the above equation) corresponding to all partitions and finally choosing the one with the highest  $M$ . However, given that state-space consisting of all the partitions is huge, thus rendering this brute force approach prohibitively expensive and infeasible. But Greedy Algorithm helps overcome this issue by systematically inspecting partitions and finding a partition corresponding to maximum  $M$ , without resorting to inspect all partitions.

**Executing Greedy Algorithm:** One of the earliest algorithm based on modularity maximization was given by Newman[38]. The essential idea is to iteratively combine pairs of communities if that step helps in raising the modularity corresponding to a partition. The algorithm in itself is very simple. Initially, each node is assigned to a community of its own, therefore, starting with  $n$  equal-sized communities. Then, it inspects every community pair that is connected with atleast one link and calculate the difference in modularity  $\Delta M$  obtained if we merge them. Essentially, the idea is to identify the community pair for which the  $\Delta M$  is the largest and merge them. It is noteworthy that modularity is a network-level statistic. Now, algorithm keeps iterating the process, until all nodes merge into one community and simultaneously storing  $M$  for each step. Finally, algorithm chooses the



partition corresponding to the highest  $M$ .

The network of communities obtained through greedy algorithm provides us with a coarser version of the network. However, with all its simplicity, adaptability and speed, Greedy Algorithm suffers from the problem of **resolution limit**[39] when applied to large networks. It essentially fails to discover well-defined small communities, present in the form of cliques. To overcome this flaw we have employed Info-map (map-equation) method to detect communities (or clusters) in our Production Network which overcomes this problem.

**Network of Communities obtained using Map Equation Method:** This method falls in the class of flow models, which tries to capture the dynamic flows on a network instead of focusing upon the topological structure of the network. For this chapter, the underlying idea is to capture the trade flows between the components of the production network. Therefore communities would then encompass all the nodes among which flow persists for a long time once entered.

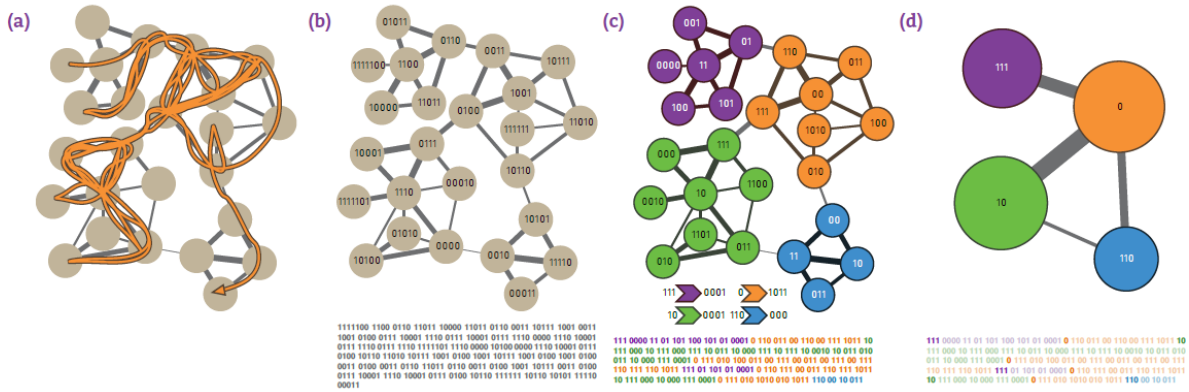
To execute the map-equation method, an algorithm called infomap is employed, which partitions the network into communities using Huffman code in such a way that compresses the description length of a random walker exploring the graph. To unpack the meaning of the sentence, one could think in terms of a random walker exploring the network, and it moves from one node to another with some probability coming from its Markov transition matrix. These probabilities could then be translated to encode the network, wherein every node is classified with an individual codeword. However, in most real-world networks, if a random walker enters a region, it tends to stay in that region for a substantial amount of time, and it rarely moves to another region. However, this whole process permits us to combinatorically combine codewords into Huffman codes, which simply requires a prefix code for each region along with a codeword uniquely assigned to each node within a module. By doing this, one can reuse node level codewords again for each module. An instructive yet straightforward analogy could be thought of as an exercise to name streets in India. It would be a daunting task to assign a unique name to every street in India. Instead, a better idea is to classify regions in states and towns, and then use street name repeatedly for different towns/states.

Then we can utilize an optimization algorithm, to reach at an optimal partition that assigns nodes to modules such that information needed to compress the movement of random

walker is minimized. For that we need to minimize the following equation:

$$L(C) = q_{\curvearrowright} H(C) + \sum_{i=1}^m p_{\circlearrowleft}^i H(P^i)$$

In the above equation,  $L(C)$  calculates the per-step mean description distance corresponding to the dynamics exhibited by a random walker while he is traveling between vertices of a network using its linkages. This holds for a given partition of the nodes  $C = C_1, \dots, C_l$ . It essentially comprises two distinct components. The first component stems from the trajectory of the random walker's movement over different communities. The symbol  $q_{\curvearrowright}$  denotes the probability of random walker switching communities, and  $H(C)$  reflects the mean description length of the code-words used to describe community index, a concept originating from Shannon-Entropy. The second component corresponds to the dynamics exhibited by the random-walker while moving within a community. For a given community  $C_i$ ,  $p_{\circlearrowleft}^i$  denotes the percentage of the movements over that community, and  $H(P^i)$  is a measure of the entropy of the code-words adapted from a module code-book  $i$ . If there are certain dense parts of the network, where random walker spends a long time, then to compress the trajectory laid by him to a short-code, we can employ a two-level encoding process by redefining nodes within that particular dense community and classifying it with a certain name. Therefore, in order to find the best community structure in a map-equation framework, we need to search for a specific partition of the nodes that minimizes the mean length of the trajectory ( $L(C)$ ) laid out by a random walker. Fig2.1 helps in understanding how the infomap algorithm undertakes this optimization problem of partitioning an actual graph into the network of communities.



(a)

Figure 2.1: Identifying communities in a network using Infomap Algorithm which compresses the movement of a random walker on a network.

Panel (a): The orange line depicts the movement of a random walker on a network. Our objective would be to efficiently describe the trajectory, essentially by assigning nodes to communities and rendering those communities unique names using minimum symbols.

Panel (b) : In this panel, the trajectory of random walker is described by giving a unique name to each node, which is estimated probability that the random walker visits that node. This method is a data compression algorithm, called Huffman coding. The 314 bits shown under the network describes the path of random walker shown in Panel (a), it starts with 1111100 for the first node in the upper left corner, where the walk originates, then second node represents 1100 and so on. The last node on the trajectory is labeled 00011, as evident in the lower right corner.

Panel (c): This panel shows that same trajectory can be compressed to a short code. For that a two level encoding-process is used to describe a random walk, in which each community receives a unique name, but the names of nodes within the community are reused. This represents 32% shorter coding. The community names along with the label used to describe an exit from each community, are shown to the left and right of the "arrow" symbol under the network. (c). For instance: initial three bits-111, represents the beginning of the walk in purple community, and 000 bits indicate the first node of the walk and so on.

Panel (d): An efficient coarse graining underlying the network is obtained by only labeling the communities with their respective names and abstaining from disclosing the position of each node within the communities.

The scheme of steps involving transformation of the entire production network to its granular representations at various levels of coarseness, is exhibited in Fig. 2.3. To give a visual idea of the resulting networks, we have sampled a small sized network consisting of 954 nodes (firms) with 1000 edges (trade linkages) among them, from the original data. Panel (a) shows extraction of the backbone via disparity filter[37]. Panels (b) and (c) exhibit the

*network of communities* constructed using modularity maximization and Infomap algorithms respectively.

### 2.2.3 First Order Degree Distributions

In Figure. 2.4, we show the degree distributions for the entire production network, the giant component (GC) and the backbone across all quarters. The  $x$ -axis denotes log of degree (in and out) and the  $y$ -axis denotes the log frequency.

As the empirical distribution suggests, there are two main features in the data. One, there is substantial heterogeneity across the firms in terms of degree distribution. The almost linear nature of the frequency plot is indicative of a fat-tailed distribution. Therefore, some firms are disproportionately more important than other firms in the buyer-seller network. Two, the in-degree and the out-degree seem to be distributionally similar to each other except that in all cases, the out-degree has a mildly sharper decay. The distributions in all the quarters have extensive overlaps.

### 2.2.4 Second-Order Degree Distribution

Second-order degree connectivity goes one step further and measures the degree of the neighbors of a firm, thereby capturing the important firms in terms of having prominent neighbors. For second-order degree distributions, we refer to Fig. 2.2 , where we have presented the results for the backbone networks for every quarter. Similar to the first-order degree distributions, second-order degree distribution also have relatively heavy tails in both the quarters. It is noteworthy in this context that Acemoglu et al.(2012)[2] showed that the fat tail in the second-order connectivity contributes to aggregate volatility. In their case, they had shown it in the context of sectoral data whereas our results utilize firm-level trade linkages.

Given the fat tailed nature of the first-order and second-order degree distributions in the original network as well as the filtered networks, we interpret the results to indicate that the topology of the network would play major role in distress propagation across the network. Disruption in one firm, who is not only supplier to a large number of firms, but also is supplier to other firms who are large suppliers, would potentially propagate from the

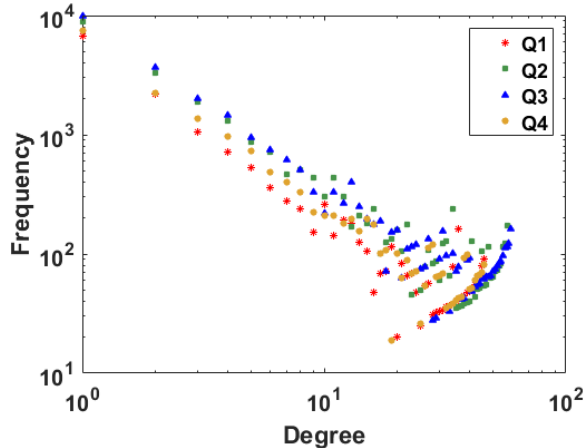


Figure 2.2: Second-order degree distributions of the backbone networks in four quarters.

epicenter to a large cluster of firms within network of distance 2.

### 2.2.5 Degree Distribution in Networks of Communities

Fig. 2.5 exhibits the degree distribution for the network of communities extracted using both modularity maximization and Infomap algorithms from the backbone of production network for two quarters. The degree distributions corresponding to the modularity maximization algorithm has much narrower range of variation than the one extracted using Infomap algorithm. This difference can be attributed to the resolution limit of the modularity maximization algorithm leading to smaller clusters. Therefore, Infomap provides a more coarse-grained clustering structure than the modularity maximization algorithm.

Table 2.3: Assortativity analysis for the coarse-grained networks

quarter	produc- tion network	GC	back- bone	com. network (mod. max.)	com. network (Infomap)
1	-0.067	-0.067	-0.19	-0.12	0.013
2	-0.07	-0.07	-0.13	-0.21	0.015
3	-0.063	-0.063	-0.14	-0.15	0.031
4	-0.062	-0.062	-0.16	-0.15	0.08

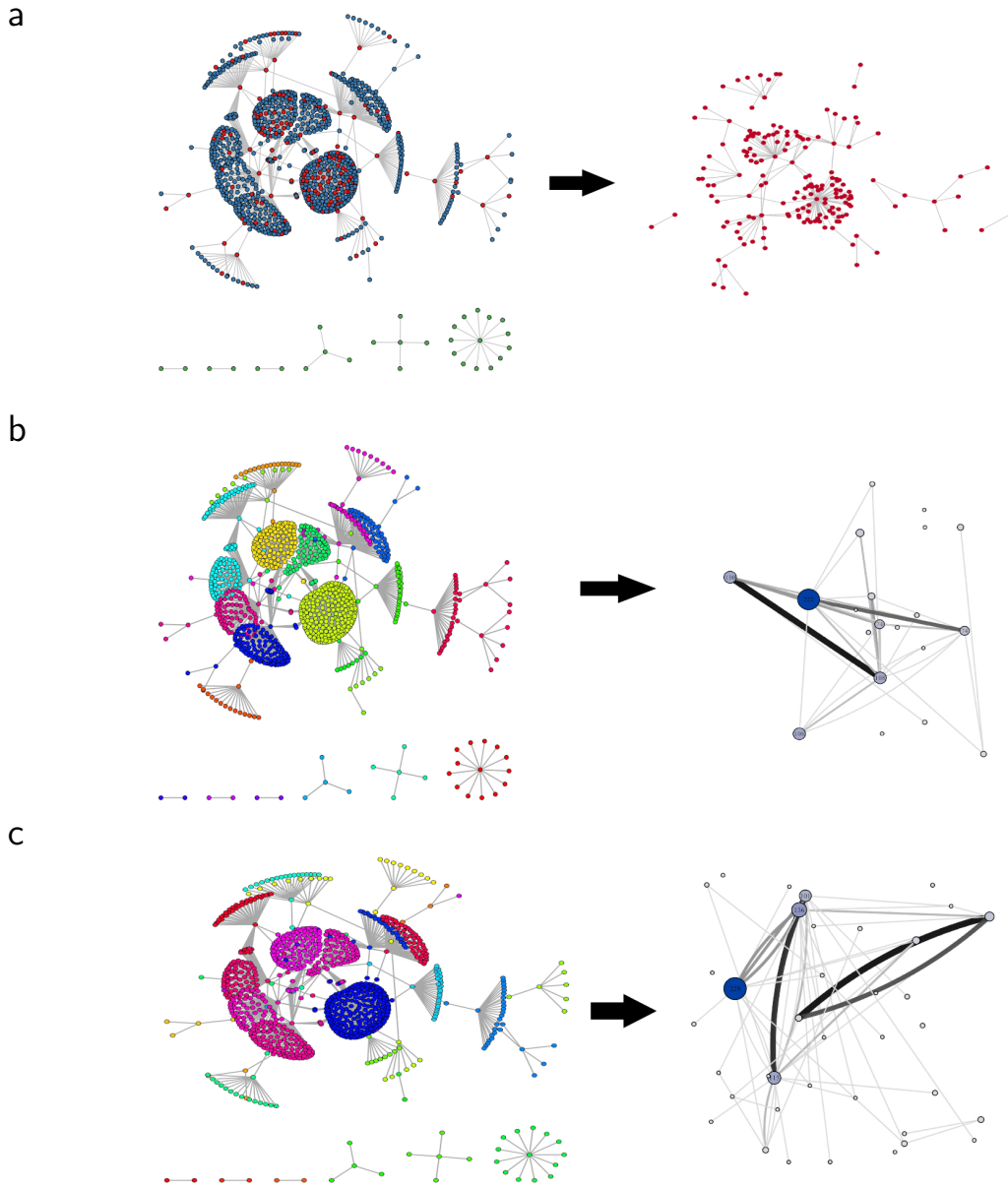


Figure 2.3: **Filtering process of the granular network: (a) extraction of the backbone from the production network, (b) community detection using modularity maximization algorithm and the resultant network of communities, and (c) community detection using Infomap algorithm and the resultant network of communities.**

Description:- Panel (a): The figure shows a sample network obtained from the data (954 nodes and 1000 edges). The network has three components – (i) giant component colored blue (925 nodes and 977 connections), (ii) unconnected peripheral nodes colored green (29 nodes with 23 edges), and (iii) the backbone (red) extracted using disparity filter[37] (171 nodes and 155 edges). In panels (b) and (c), we show the *network of communities* extracted from the production network. Each community is comprised of a set of firms and is collapsed into one node in the *network of communities*. The size of each node is scaled according to the number of firms in it. The edges are weighted according to the number of connections between the constituent firms belonging to these communities. Panel (b): 20 communities are extracted using modularity maximization algorithm. The largest community has 222 firms in it. Panel (c): 38 communities extracted using Infomap algorithm. The largest community has 229 firms in it.

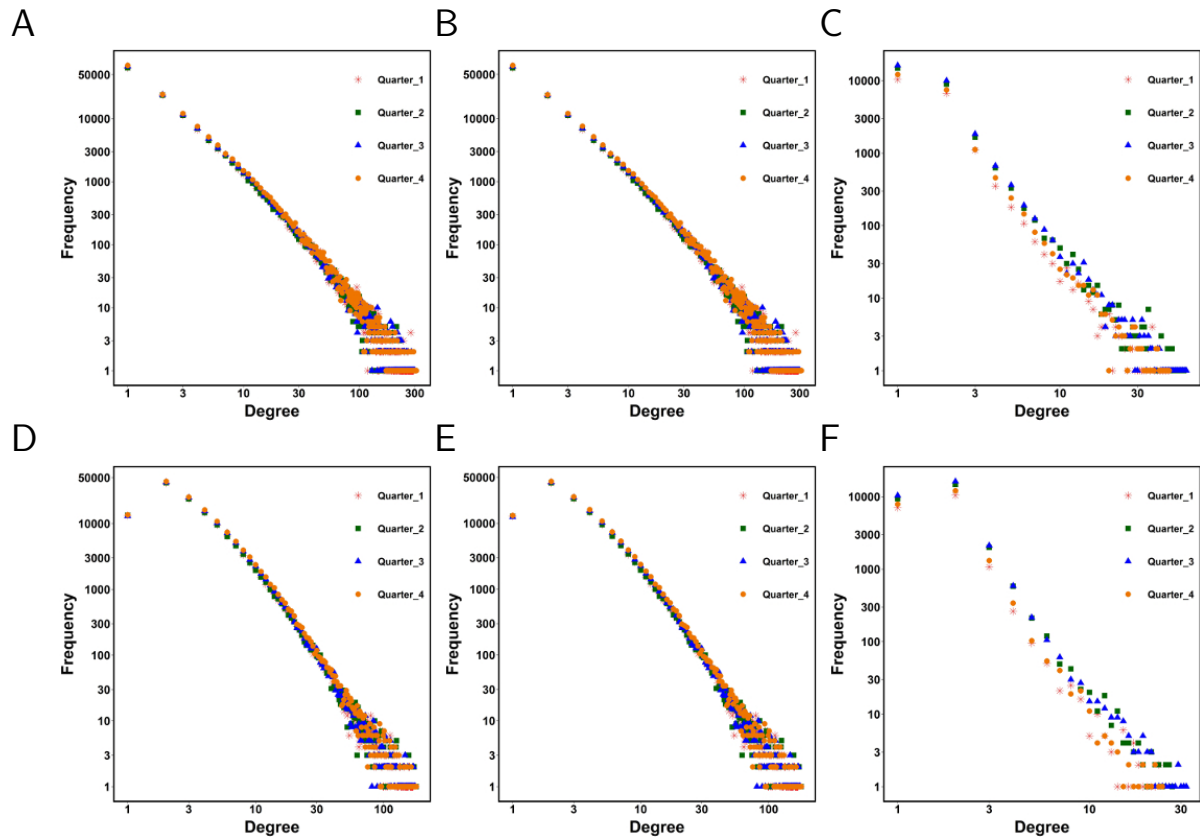


Figure 2.4: **In-degree and out-degree distributions.** Plots in log-log scale for the four quarters in the fiscal year 2016. Upper panel shows in-degree distribution and Lower panel shows out-degree distribution: (a) and (d) Entire production network, (b) and (e) Giant component, (c) and (f) Backbone. In all three cases, the distributions are quite stable even after the shock of *demonetisation* between the second and third quarters. Out-degree seems to have exhibit a slightly steeper decay than in-degree in all cases, indicating lesser degree of heterogeneity.

**Definition 2.2.2.** *Assortativity of a Network:* The assortativity coefficient is the Pearson correlation coefficient of degree between pairs of linked nodes. It can be calculated by the following expression:

$$assortativity(g) = \frac{\sum_{i,j \in g} (d_i - m)(d_j - m)}{\sum_{i \in N} (d_i - m)^2}$$

$m$  : Average degree of  $g$

$d_i$  : degree of node  $i$

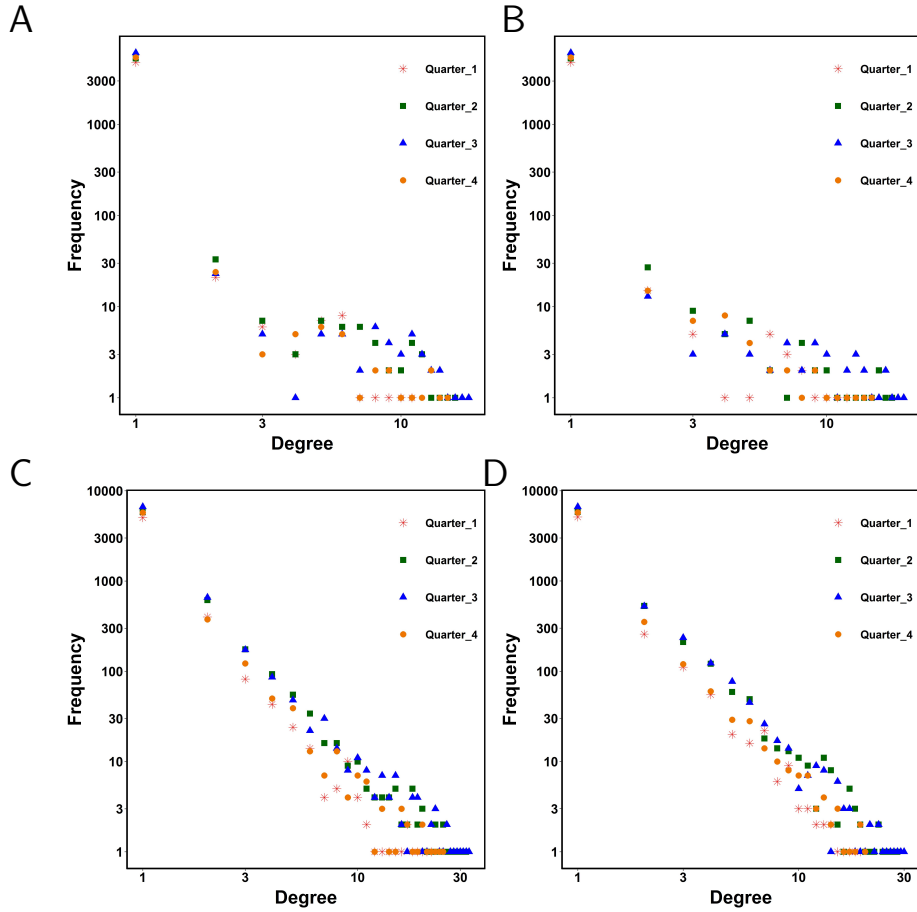


Figure 2.5: Upper-Panel-A(B) shows In(Out)-Degree distribution of the modularity maximization community network in four quarters. (b) Lower-Panel-C(D) shows In(Out)-Degree distribution of the Infomap community network.

In Table 2.3, we report the assortativity coefficients (defined in Def 2.2.2) of the networks at different levels of granularity. We see that the entire network along with the giant component, are mildly disassortative. However, the backbone network is quite strongly disassortative. the community networks obtained through modularity maximization are strongly disassortative whereas the community network obtained from Infomap algorithm is very mildly assortative in nature. The disassortative nature of the network of communities (along with large heterogeneity in degree distribution) is important in the context of shock propagation, since it indicates that there are multiple hub-and-spoke structure embedded in the network where the hubs are indirectly connected. Therefore, shocks generated in one hub can transmit to other hubs and the corresponding spokes. Below, we formally address the phenomena of shock propagation in the context of a dynamical model.



## 2.3 Modularity and Shock Propagation in Network of Communities

In order to model shock propagation in an interlinked economy, we rely on a simple statistical dynamic model called vector autoregression model (VAR). This model allows us to calibrate the interlinkages to the observed empirical network and thereby uniquely identify a path for shock propagation from any chosen epicenter to all other nodes in the network.

The  $VAR(p)$  framework represents the evolution of  $n$ -dimensional time series  $x_t$  as a function of their lagged values of the order  $1 \leq \dots \leq p$  along with cross-dependence and a vector of error terms.

$$\begin{bmatrix} x_{1t} \\ \vdots \\ x_{nt} \end{bmatrix} = \begin{bmatrix} a_{11}^1 & \dots & a_{1n}^1 \\ \vdots & \ddots & \\ a_{n1}^1 & & a_{nn}^1 \end{bmatrix} \cdot \begin{bmatrix} x_{1t-1} \\ \vdots \\ x_{nt-1} \end{bmatrix} + \dots + \begin{bmatrix} a_{11}^p & \dots & a_{1n}^p \\ \vdots & \ddots & \\ a_{n1}^p & & a_{nn}^p \end{bmatrix} \cdot \begin{bmatrix} x_{1t-p} \\ \vdots \\ x_{nt-p} \end{bmatrix} + \begin{bmatrix} u_{1t} \\ \vdots \\ u_{nt} \end{bmatrix} \quad (2.1)$$

or, in matrix form

$$x_t = A_1 x_{t-1} + \dots + A_p x_{t-p} + u_t \quad (2.2)$$

where  $x_t = (x_{1t}, \dots, x_{nt})$  denote  $n$ -dimensional time series,  $A_k$  denotes  $(n \times n)$  coefficient matrix with elements  $a_{ij}^k$  where  $k$  corresponds to the lag order,  $i$  and  $j$  are row and column index respectively,  $u_t = (u_{1t}, \dots, u_{nt})$  is a  $n$ -dimensional error vector with mean  $\mathbf{E}(u_t) = \mu$  and covariance matrix  $\mathbf{E}(u_t u_t^T) = \Sigma_u$ .

## Application of VAR: Shock Propagation on Network of Communities

The essential idea behind applying VAR in the present context is that we can imagine, that each firm has its own idiosyncratic shock process (fluctuations due to local demand and supply factors) and due to the interlinked nature of the shock process, one firm's shock gets transmitted to its first-order neighbors and then their first-order neighbors and so on. In

order to implement the mechanism, we first impose a simple structure on the interlinkages:

$$x_t = Ax_{t-1} + u_t, \quad (2.3)$$

where we have deliberately chosen a simple model with one lag ( $p = 1$  in Eqn. 2.2) to reduce dimensionality of the model and parameters. The matrix  $A$  gives us the connectivity structure across the firms, which can be mapped to the empirical data.

In actual implementation, we calibrate the matrix  $A$  to the adjacency matrix of the network of communities rather than the network of firms. The main reason for doing that is to reduce computational burden. Dealing with a system of equation with 0.14 million variables would be prohibitively expensive in terms of computation. The second reason is that whereas purely microeconomic shock to an individual firms may not have a substantial impact, a cluster-wide shock will surely have much more sizeable effect. We utilize *impulse response functions* to measure the impact and propagation of a single idiosyncratic shock, across the network. We select one epicenter node and give a shock to it in a network through vector  $u$ . Then by iterating the VAR equation, we can capture the dynamic evolution of resulting propagation of the shock over time and across nodes. In order to establish that different topology of the networks lead to different patterns of distress propagation, we will first demonstrate the mechanism based on four models depicting rudimentary network structures.

In Fig. 2.6, we provide the network structures with the epicenters of distress (top panels; epicenter has darker shade) and the corresponding evolution of distress across the network (bottom panels). In panel (a), we have a circular network where each node is connected with its two immediate neighbors. Panel (b) shows a star-shaped core-periphery network. Panel (c) shows a complete graph. Panel (d) shows a linear graph. All networks have exactly four nodes for the sake of comparison. In the bottom panels, we provide the impulse response functions corresponding to each node (nodes 1 to 4) over 10 time points. The intensity of the shock is captured by the height of the impulse response function (color coding shown by the side of the figures).

The nature of the shock propagation is quite intuitive. In panel (a), Node 1 is the epicenter which directly connects to Nodes 2 and 4. Both Nodes 2 and 4 are recipients of the shock but Node 3 which is in distance 2, does not react substantially (bottom panel). In panel (b), a shock to the core node (Node 1) equally impacts all the peripheral nodes.

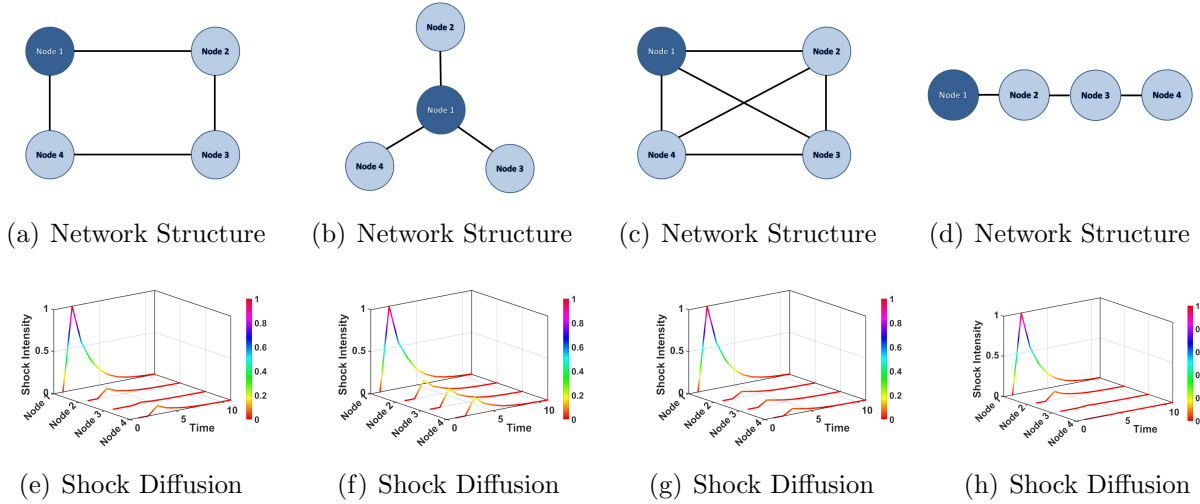


Figure 2.6: **Distress propagation in toy models.** The plots show the usage of VAR(1) model to demonstrate the varying effect of different network structures on distress propagation through the network over time.

For a complete graph (panel (c)), all nodes are symmetric and hence receives the shock in equal intensity. In panel (d), we have a linear graph and the epicenter is Node 1 which is the left-most node. As the impulse response functions in the bottom panels indicate, the shock propagates to other nodes but with lesser and lesser intensity. Node 4 (which is in distance 3 from the epicenter) shows almost no response. The main take away from this simple exercise is that a core-periphery structure is the most conducive for distress propagation in an asymmetric manner (periphery to core shock propagation can be shown to have mild impact and periphery to periphery through core would be almost negligible).

## 2.4 Distress propagation: Results and Discussions

In this section, we implement the VAR model on the empirical network of communities. As we have described above, we calibrate the interaction matrix to the adjacency matrix of the observed network. Then we choose an epicenter and give a shock to it. The resultant impulse response functions allow us to study distress propagation over time and across the network.

First, we note that for each quarter there are two types of networks of communities; the first one is constructed by applying the modularity maximization algorithm, the second

one is constructed by applying the Infomap algorithm. We have already noted before that the network of communities obtained from the Infomap algorithm is more coarse-grained than the one obtained from modularity maximization algorithm (see Fig. 2.5). Also, it is worthwhile to note from table 2.3 that the network of communities obtained from the Infomap algorithm is substantially less disassortative (very mildly assortative) than the one obtained from modularity maximization algorithm.

In Fig. 2.7, we have plotted the network of communities constructed via modularity maximization algorithm in quarter 1-4 in the fiscal year 2016. And Fig.2.8, shows distress propagation on the same network of communities. Panel (a)-(d) in Fig.2.8 shows the network of communities. We give a unit shock to the largest node (denoting the largest community) and record the response across all nodes over time, as shown in panel (a),(c),(e),(g) of Fig2.8. We see that a large number of nodes respond after one to three periods and the peak response is about 20% of the original shock (the  $z$ -axis denotes the shock intensity; colorbar shown in the figure). Following an alternate approach, we studied distress diffusion on the networks of communities where the epicenter of the shocks are the nodes with highest eigenvector centrality.[2, 40] The results are presented in panel-(b),(d),(f),(h) of Fig. 2.8.

We implement the same exercise using network of communities that have been constructed via Infomap algorithm. The structure of community networks obtained through Infomap algorithm is shown in Fig2.9. The results of distress propagation corresponding to those networks are shown in Fig. 2.10. Again we give a unit shock to the largest community, and record the impulse response across all nodes over time, as shown in panel (a),(c),(e),(g) of Fig.2.10. One of the notable difference from the earlier result (i.e. the network of communities based on modularity maximization) is that here the effects are much more heterogeneous; fewer number of nodes respond and the response intensity is also comparatively smaller in magnitude. We also conduct the same analysis here when we give shock to node with the highest eigen-vector centrality in case of infomap-community network and record responses in the same way. The results are displayed in panel-(b),(d),(f),(h) of Fig. 2.10. The main observation is that the responses generated on the Infomap modular networks are much more muted and the extent of shock propagation is limited.

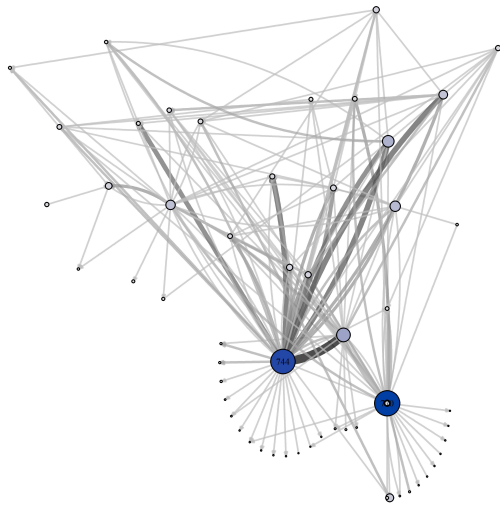
Findings are both qualitatively and quantitatively similar in all the concerned diagrams. Note that production clusters are nothing but a group of firms that are well-integrated in terms of supply-chain. The weight of a trade linkage from one production cluster to

another signifies the normalized version of the total number of trade linkages emanating from the firms in that production cluster to another. Therefore, big production clusters have more prominent connections because there are more firms (and therefore linkages) in that cluster compared to even a central production community, and hence the size of the community matters in amplifying the shock and not its position in the context of the network of production clusters. Also, there can be multiple central firms (general-purpose input suppliers to a lot of essential firms) within each supply chain. Notably, the propagation patterns are susceptible to the choice of epicenters in the network of communities. Shocks to the largest communities impact the system more than the community with the highest centrality. This finding indicates that ‘too-big-to-fail’ might be more dominant than ‘too-central-to-fail’ entities. The result is robust to the choice of any other arbitrary epicenter. Propagation patterns are rich when the biggest community is subjected to shock, and this phenomenon is not evident for any other choice of the epicenter. (one such choice of highest eigenvector centrality has been shown)

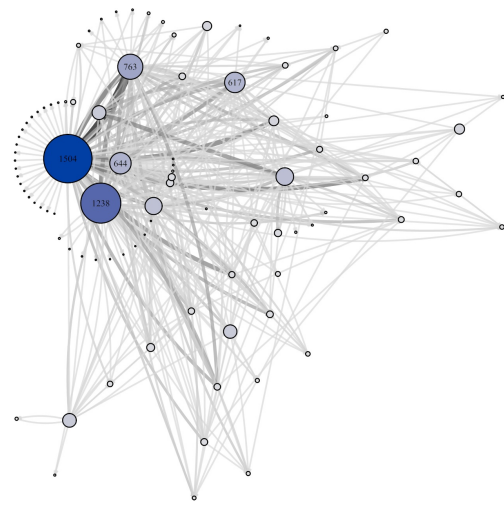
Therefore, in this section we have established that the nature of coarse-grained modular structure of the network has important implications for the mechanism of shock propagation. A more disassortative network structure (obtained through modularity maximization;2.3) leads to wider diffusion of shocks and the dynamic responses of the nodes are also larger in magnitude. We also see that ‘too-big-to-fail’ modules seem to be more prominent for shock diffusion than ‘too-central-to-fail’ modules in terms of impacting the network.

## 2.5 Summary and Discussion

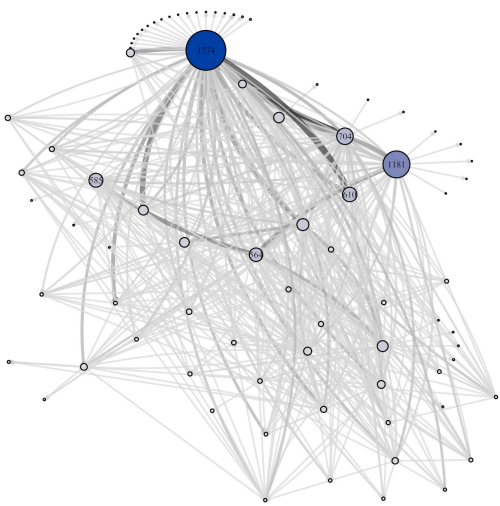
Distress propagation on complex networks is a dynamic process that is known to be affected by the topology of the underlying networks. The recent literature in economics and finance recognizes the role of the network architecture in the phenomena of economy-wide fluctuations.[2][41][3][31] However, one major problem is the lack of granular data to quantify the network and analyze the mechanism of shock diffusion. In this article, we leverage an unique administrative dataset obtained from the Indian state of West Bengal that allows us to construct firm-to-firm buyer-seller network at the smallest level of granularity by matching the tax records produced for each transaction by both parties of the transaction, i.e. both the buying firm and selling firm report the transaction. Therefore, by matching the records



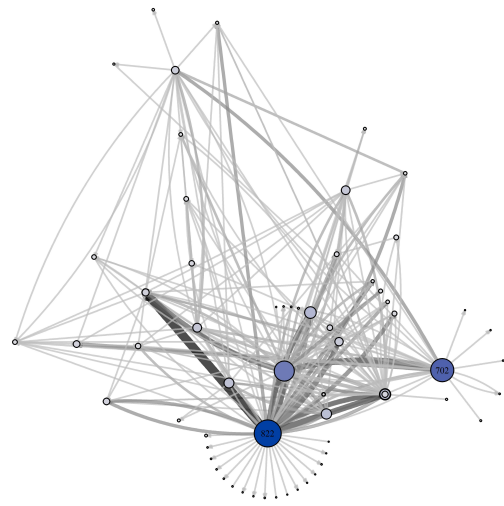
(a) Quarter 1



(b) Quarter 2



(c) Quarter 3



(d) Quarter 4

Figure 2.7: **Networks of the communities (modularity maximization algorithm) from backbone networks for quarters 1-4.** Communities are depicted as nodes and their sizes are scaled to the number of constituent firms. We have labeled nodes (communities) with at least 600 firms. While constructing the network, we have removed self loops and nodes with degree zero. Panel (a): Q1- 58 nodes and 166 edges. Panel (b): Q2- 93 nodes and 389 edges. Panel (c): Q3- 79 nodes and 447 edges. Panel (d): Q4- 65 nodes and 225 edges.

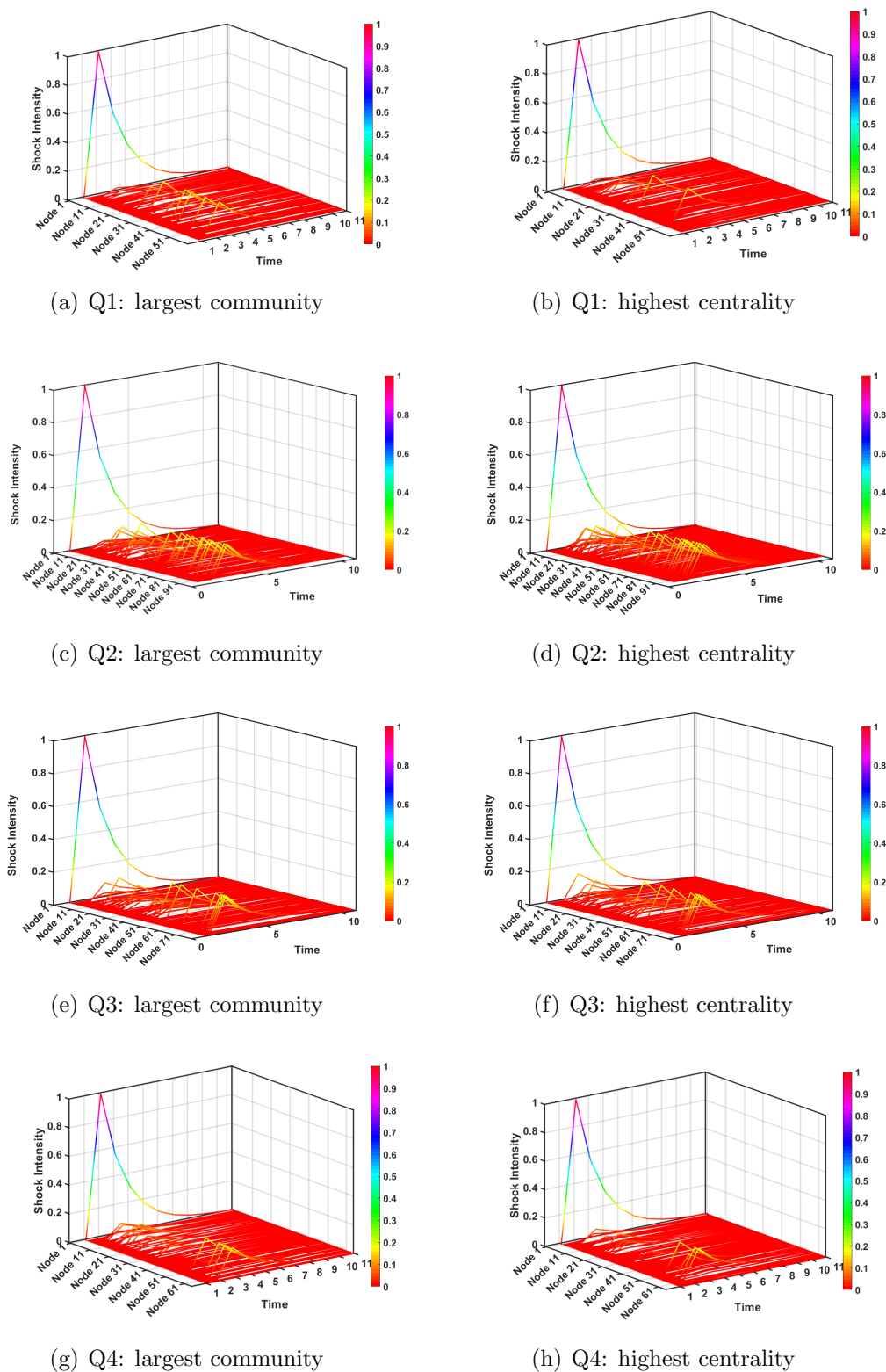


Figure 2.8: **Impulse response functions corresponding to networks in Fig. 2.7 for quarters 1-4.** Panels (a, c, e, g): Epicenters are the largest communities in respective networks. Panels (b, d, f, h): Epicenters are the communities with largest centrality in respective networks. As evident, distress propagation initiated from the largest communities create more impact than distress initiated from the communities with highest centrality.

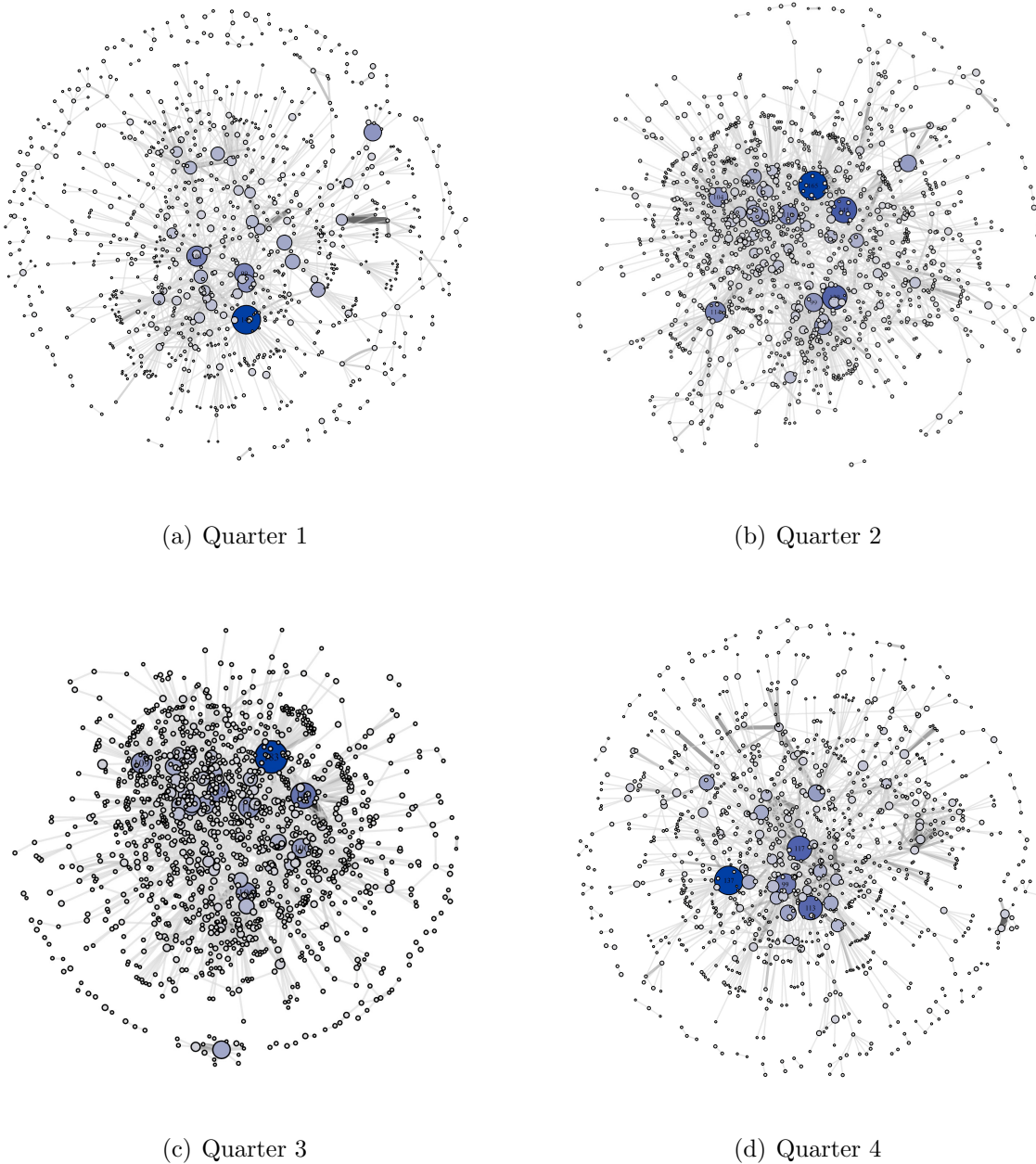


Figure 2.9: **Networks of the communities (Infomap) from backbone networks for quarters 1-4.** Communities are depicted as nodes and their sizes are scaled to the number of constituent firms, after removing self-loops and nodes with degree 0. We have labeled nodes (communities) with at least 600 firms:- Panel (a): Q1- 909 nodes and 1316 edges. Panel (b): Q2- 1257 nodes and 2538 edges. Panel (c): Q3- 1343 nodes and 2718 edges. Panel (d): Q4- 1027 nodes and 1639 edges.



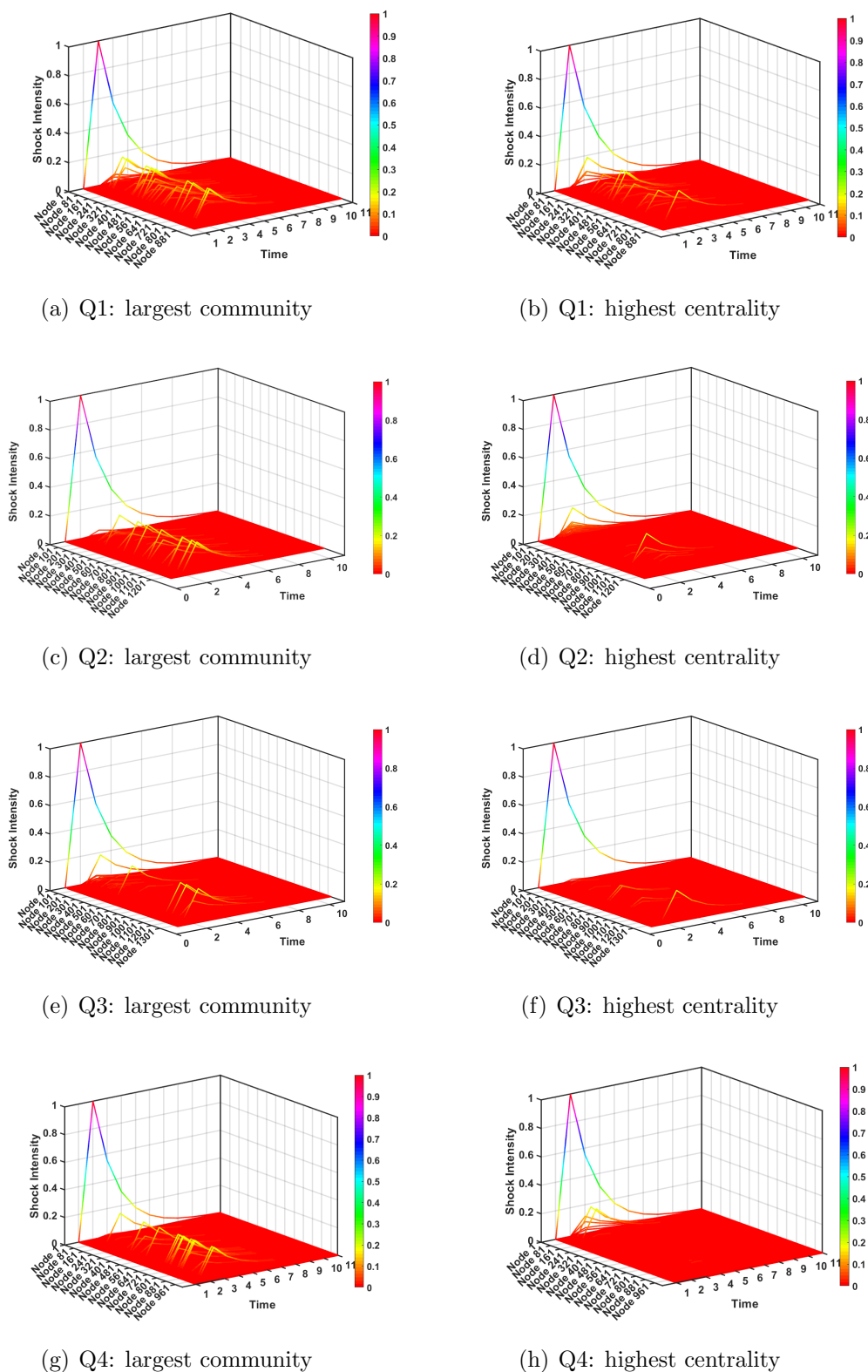


Figure 2.10: **Impulse response functions corresponding to networks in Fig. 2.9 for quarters 1-4.** Panels (a, c, e, g): Epicenters are the largest communities in respective networks. Panels (b, d, f, h): Epicenters are the communities with largest centrality in respective networks. As evident, distress propagation initiated from the largest communities create more impact than distress initiated from the communities with highest centrality. In quarter 3, the spillover mechanism is muted compared to the rest of the quarters.

one can create a bilateral linkage between two firms and by collating all such linkages arising out of transactions within a given period of time (such data is collected every quarter), we have created the complete network of transactions across all the registered firms. There are around 0.14 million firms in the database with more than 0.50 million reported linkages every quarter. Our data spans from the first to fourth quarter in the fiscal year 2016.

The main findings can be categorized under three heads. One, the network shows large heterogeneity in degree connectivity, both in the first-order and second-order.[2] Two, The coarse-grained networks of communities, created by applying modularity maximization algorithm exhibits moderate disassortativity with strong presence of multiple hub-and-spoke structures, whereas Infomap algorithm generates a modular structure with very mild assortativity. Three, the nature of distress propagation was substantially different across the networks of communities, viewed at different levels of coarseness. In particular, the more disassortative, coarser networks gave rise to network-wide diffusion of shocks and the magnitude of the diffused shock was large; the more assortative, less granular network led to muted diffusion of shocks with a limited region of spread. Our simulation results on the empirical coarse-grained structures indicate that ‘too-big-to-fail’ nodes are more dominant than ‘too-central-to-fail’ nodes in terms of distress propagation, which is a significant result in the context of identifying the relative contribution of size vis-a-vis centrality to network fragility.

The findings complement the current literature on diffusion processes on complex networks. The uniqueness of the data provides an unprecedented view into the topological structure of production networks in the context of developing countries. Some prior studies have analyzed the data from developed countries like Japan.[42][43][44] We note that there is an important difference between the production structure in Japan vis-a-vis countries like India. The first one is known to be dependent on just-in-time production process relying on very fast and efficient supply chain, whereas in case of India (can be considered to be representative of the developing countries in terms of production process), the supply chains are much less efficient and therefore, firms depend on buffer stocks. The macroeconomic implication is that a disruption in such a network might possibly take more time to diffuse through the network and therefore, there would be lagged effects on the network. This is consistent with the stickyness we find in the network structure before and after the demonitization shock whereby the government declared the existing cash to be invalid overnight. Our results indicate that at least at the macro-level, the production network did not change

substantially.

Finally, we would like to mention that there are two limitations of our present study about the mechanisms of distress propagation: (i) The network we studied belongs to one state within a federal union of India with 28 states and 8 union territories. Thus the observed network is a small subset of a larger network of firms across states (or even countries, if one consider international trade). Therefore, shocks emanating from one state, in principle, could be transmitted to the network we have studied via trading partners within West Bengal. This is amenable in our framework, where instead of individual communities, we would need to give shocks to a set of firms (affected by exogenous shocks) and simulate the shock diffusion processes. However, such data is currently not available to us. Furthermore, the computation over a country-wide firm-firm network (even without considering international linkages) would be immensely costly. (ii) We have not studied how such connections across firms had formed initially.[45] Given the data limitations, it is not possible to conduct such an analysis. In other words, we have taken the network structure as given[12][21] and studied the process of distress propagation on the network. In a complex world of evolving interconnections[46][47][31], it would be interesting to find how the network evolves and accordingly how the shock diffusion process alters[48]. Lastly, the analysis carries out in the chapter had limited economic relevance. A possible extension in that respect is discussed in the next chapter, wherein we explore the possibility of mapping the production network to a structural macroeconomic model to conduct a volatility decomposition analysis at an economy-wide level.

## **2.6 Future Extension: Structural Factor Macro-Model at the Firm Level**

Concerning this rapidly emerging literature, one meaningful direction of progress would be to conduct the analysis shown in section 5 of the chapter1 but using more granular entities than sectors. In particular, an exciting extension would be to carry out the structural factor analysis of aggregate vs. idiosyncratic shocks, using the plant or firm-level input-output network specifically in case of a developing economy like India. Although, not plenty, but some progress has been in this direction. For instance: Grassi(2017)[49] using the framework of an interlinked oligopolistic market, shows that 34% of the volatility observed

at an aggregate level is attributable to the firm-level idiosyncratic shocks. On a similar vein, Magerman(2017)[50] & Kikkawa(2019)[51] leverages the richness of extensive Belgium VAT data-set to build a firm-level production network and then calibrate a structural model using it. They provide a shred of startling evidence that firm-level idiosyncratic shocks account for almost 57% of the share in aggregate volatility of output growth in Belgium. That shows, a more granular version of the production network, composed of firm-level interlinkages, could be used as a framework to trace the propagation of shocks emanating from firm-level supply chains to economy-wide fluctuations.

The analysis carried out in this chapter was restrictive in its scope. In particular, the analysis carried out here was generally on the lines of partial equilibrium analysis. We tried to characterize the shock propagation process in terms of the topology of the community level production network but refrained from attempting to present a holistic view of the propagation patterns on different economic variables. For instance: A VAR model based on a structural macroeconomic model could have helped in capturing the dynamics of different economic variables like consumption, investment, etc. induced by the production linkages in an economy when calibrated at the cluster level. The main reason for not conducting the structural analysis was the unavailability of extensive data on time series related to production, investment, and other economic variables in that context. Nevertheless, without any doubt, the general equilibrium analysis of the dynamics induced by firm-level interlinkages in an economy is imperative. It would shed light on how different shocks lead to distinct propagation patterns in the economy. Although the atheoretical VAR model deployed in this chapter is dynamical, however, it has some limitations. It does not capture how shocks would have interacted with different economic variables and resulted in variability in subsequent periods. For instance: It is highly plausible that reduction in demand owing to a negative regional/local shock-like strikes or power-cut originating from a cluster of firms, gets entirely offset by the surge in demand induced by a positive productivity shock in some other cluster of the firms and therefore variability in the overall economy remains constant despite those disruptions. Although, the salience of the model in chapter-2 lies in identifying and quantifying the role of distinct network structures in propagating and amplifying shocks originating from one node to distant parts of the network, nevertheless, to capture rich dynamics as stated in the above example lies outside the purview of the model. The structural model discussed in the section 5.3 of chapter1 fulfills that deficiency, as it qualifies to be a general equilibrium model and is relatively dynamic to capture the propagation patterns of idiosyncratic shocks across the network.

# Bibliography

- [1] John B Long Jr and Charles I Plosser. Real business cycles. *Journal of political Economy*, 91(1):39–69, 1983.
- [2] Daron Acemoglu, Vasco M Carvalho, Asuman Ozdaglar, and Alireza Tahbaz-Salehi. The network origins of aggregate fluctuations. *Econometrica*, 80(5):1977–2016, 2012.
- [3] Daron Acemoglu, Ufuk Akcigit, and William Kerr. Networks and the macroeconomy: An empirical exploration. *NBER Macroeconomics Annual*, 30(1):273–335, 2016.
- [4] Vasco M Carvalho, Makoto Nirei, Yukiko Saito, and Alireza Tahbaz-Salehi. Supply chain disruptions: Evidence from the great east Japan earthquake. *Columbia Business School Research Paper*, -(17-5), 2016.
- [5] David Rezza Baqaee and Emmanuel Farhi. Macroeconomics with heterogeneous agents and input-output networks. Technical report, National Bureau of Economic Research, 2018.
- [6] Charles R Hulten. Growth accounting with intermediate inputs. *The Review of Economic Studies*, 45(3):511–518, 1978.
- [7] Xavier Gabaix. The granular origins of aggregate fluctuations. *Econometrica*, 79(3):733–772, 2011.
- [8] Vasco Carvalho and Xavier Gabaix. The great diversification and its undoing. *American Economic Review*, 103(5):1697–1727, 2013.
- [9] Saki Bigio and Jennifer La’o. Financial frictions in production networks. *NBER working paper*, -(w22212).
- [10] David Rezza Baqaee and Emmanuel Farhi. Productivity and misallocation in general equilibrium. *The Quarterly Journal of Economics*, 135(1):105–163, 2020.
- [11] Basile Grassi et al. Io in io: Size, industrial organization, and the input-output network make a firm structurally important. Technical report.

- [12] Enghin Atalay, Ali Hortacsu, James Roberts, and Chad Syverson. Network structure of production. *Proceedings of the National Academy of Sciences*, 108(13):5199–5202, 2011.
- [13] Vasco M Carvalho and Nico Voigtländer. Input diffusion and the evolution of production networks. Technical report, National Bureau of Economic Research, 2014.
- [14] Matthew O Jackson and Brian W Rogers. Meeting strangers and friends of friends: How random are social networks? *American Economic Review*, 97(3):890–915, 2007.
- [15] Ezra Oberfield. A theory of input–output architecture. *Econometrica*, 86(2):559–589, 2018.
- [16] Mathieu Taschereau-Dumouchel. Cascades and fluctuations in an economy with an endogenous production network. *Available at SSRN 3115854*, 2019.
- [17] Robert E Lucas Jr. Understanding business cycles. In *Carnegie-Rochester conference series on public policy*, volume 5, pages 7–29. North-Holland, 1977.
- [18] Jean-Noël Barrot and Julien Sauvagnat. Input specificity and the propagation of idiosyncratic shocks in production networks. *The Quarterly Journal of Economics*, 131(3):1543–1592, 2016.
- [19] Christoph E Boehm, Aaron Flaaen, and Nitya Pandalai-Nayar. Input linkages and the transmission of shocks: firm-level evidence from the 2011 tōhoku earthquake. *Review of Economics and Statistics*, 101(1):60–75, 2019.
- [20] Banu Demir, Beata Javorcik, Tomasz K Michalski, and Evren Ors. Financial constraints and propagation of shocks in production networks. *Work. Pap., Univ. Oxford, UK*, 2018.
- [21] Andrew T Foerster, Pierre-Daniel G Sarte, and Mark W Watson. Sectoral versus aggregate shocks: A structural factor analysis of industrial production. *Journal of Political Economy*, 119(1):1–38, 2011.
- [22] John B Long and Charles I Plosser. Sectoral vs. aggregate shocks in the business cycle. *The American Economic Review*, 77(2):333–336, 1987.
- [23] Mario Forni and Lucrezia Reichlin. Let’s get real: a factor analytical approach to disaggregated business cycle dynamics. *The Review of Economic Studies*, 65(3):453–473, 1998.
- [24] John Shea. Complementarities and comovements. *Journal of Money, credit and Banking*, pages 412–433, 2002.
- [25] Michael TK Horvath. Cyclicity and sectoral linkages: Aggregate fluctuations from independent sectoral shocks. *Available at SSRN 343*, 1997.
- [26] Michael Horvath. Sectoral shocks and aggregate fluctuations. *Journal of Monetary Economics*, 45(1):69–106, 2000.

- [27] Bill Dupor. Aggregation and irrelevance in multi-sector models. *Journal of Monetary Economics*, 43(2):391–409, 1999.
- [28] Vasco M Carvalho. Aggregate fluctuations and the network structure of intersectoral trade. *NBER Working Paper*, .(.):., 2007.
- [29] Enghin Atalay. How important are sectoral shocks? *American Economic Journal: Macroeconomics*, 9(4):254–80, 2017.
- [30] Ashish Kumar, Anindya S. Chakrabarti, Anirban Chakraborti, and Tushar Nandi. Distress propagation on production networks: Coarse-graining and modularity of linkages, 2020.
- [31] Michele Starnini, Marián Boguñá, and M Ángeles Serrano. The interconnected wealth of nations: Shock propagation on global trade-investment multiplex networks. *Scientific reports*, 9(1):1–10, 2019.
- [32] N Gregory Mankiw. Real business cycles: A new keynesian perspective. *Journal of Economic Perspectives*, 3(3):79–90, 1989.
- [33] Michael Horvath. Cyclicalities and sectoral linkages: Aggregate fluctuations from independent sectoral shocks. *Review of Economic Dynamics*, 1(4):781–808, 1998.
- [34] H David, David Dorn, and Gordon H Hanson. The China syndrome: Local labor market effects of import competition in the United States. *American Economic Review*, 103(6):2121–68, 2013.
- [35] Mark Ed Newman, Albert-László Ed Barabási, and Duncan J Watts. *The structure and dynamics of networks*. Princeton university press, 2006.
- [36] Albert-László Barabási et al. *Network science*. Cambridge university press, 2016.
- [37] M Ángeles Serrano, Marián Boguñá, and Alessandro Vespignani. Extracting the multiscale backbone of complex weighted networks. *Proc. Natl. Acad. Sci.*, 106(16):6483–6488, 2009.
- [38] Mark EJ Newman. Fast algorithm for detecting community structure in networks. *Physical Review E*, 69(6):066133, 2004.
- [39] Santo Fortunato and Marc Barthelemy. Resolution limit in community detection. *Proceedings of the National Academy of Sciences*, 104(1):36–41, 2007.
- [40] Abhijit Banerjee, Arun G Chandrasekhar, Esther Duflo, and Matthew O Jackson. The diffusion of microfinance. *Science*, 341(6144):1236498, 2013.
- [41] Daron Acemoglu, Asuman Ozdaglar, and Alireza Tahbaz-Salehi. Systemic risk and stability in financial networks. *American Economic Review*, 105(2):564–608, 2015.

- [42] Abhijit Chakraborty, Yuichi Kichikawa, Takashi Iino, Hiroshi Iyetomi, Hiroyasu Inoue, Yoshi Fujiwara, and Hideaki Aoyama. Hierarchical communities in the walnut structure of the japanese production network. *PloS one*, 13(8):e0202739, 2018.
- [43] Yoshi Fujiwara and Hideaki Aoyama. Large-scale structure of a nation-wide production network. *The European Physical Journal B*, 77(4):565–580, 2010.
- [44] Takashi Iino and Hiroshi Iyetomi. Community structure of a large-scale production network in japan. In *The Economics of Interfirm Networks*, pages 39–65. Springer, 2015.
- [45] Angelo Mele. A structural model of dense network formation. *Econometrica*, 85(3):825–850, 2017.
- [46] Daron Acemoglu, Ufuk Akcigit, and William R Kerr. Innovation network. *Proceedings of the National Academy of Sciences*, 113(41):11483–11488, 2016.
- [47] W Brian Arthur. Complexity and the economy. *Science*, 284(5411):107–109, 1999.
- [48] Mark EJ Newman. Spread of epidemic disease on networks. *Physical Review E*, 66(1):016128, 2002.
- [49] Vasco M Carvalho and Basile Grassi. Large firm dynamics and the business cycle. *American Economic Review*, 109(4):1375–1425, 2019.
- [50] Andrew B Bernard, Emmanuel Dhyne, Glenn Magerman, Kalina Manova, and Andreas Moxnes. The origins of firm heterogeneity: A production network approach. *National Bank of Belgium, Working Paper No. 362*, 2019.
- [51] Ken Kikkawa, Glenn Magerman, and Emmanuel Dhyne. Imperfect competition in firm-to-firm trade. *Available at SSRN 3389836*, 2019.